The relationship between various analytical techniques of T-duality

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Introduction

String theory

A theory which regards elementary objects as a string

- closed string
- open string
- gauge field $A_a$

String theory is a candidate of quantum gravity, but defined only perturbatively
- we want to find the stringy geometry
  (or: general relativity $\leftrightarrow$ Riemannian geometry)
- This must encode the property of T-duality

T-duality

Spectrum of the closed string

$2\pi R$ vs. $\frac{1}{\alpha^{'}}$

D-brane

the hypersurface where open strings end.

- direction along the D-brane
  - Neumann condition

- direction perpendicular to the D-brane
  - Dirichlet condition

The effective theory on N D-branes
- $U(N)$ gauge theory

T-duality and D-brane

Dirichlet and Neumann condition are exchanged under the T-duality

- T-dual normal to the D-brane
  - $Dp \rightarrow D(p + 1)$

- T-dual along the D-brane
  - $Dp \rightarrow D(p - 1)$

Boundary state of D-branes

Bound state of the D-branes

- Correspond to a soliton solution in the effective theory
- Topological charge $=$ the number of D-branes

Various approaches to discuss T-duality

- Buscher rule
- Nahm transformation
- Boundary state analysis
- Hori formula

Nahm transformation

Original Nahm transformation

$\mathcal{T}(N)$ Gauge theory on $T^2$

$U(k)$ instanton on $\mathbb{C}^2$

D-brane interpretation

$N \mathcal{D}_k / k \mathcal{D}_k$

Generalization of Nahm transformation to 2d

We consider the classical solution on $T^2$

- constant magnetic flux

There is a freedom to add constants $\beta_1, \beta_2, \gamma_1, \gamma_2$

We will construct the gauge field on $T^2$ by

Dirac zero mode $(D_k \psi = 0)$

$D_k \psi = 0$ has $k$ solutions

D-brane transformation

$N \mathcal{D}_k / k \mathcal{D}_k \rightarrow k \mathcal{D}_k / N \mathcal{D}_k$

Fermionic Part

NS-NS sector $(\sigma, \phi) (x^{\mu}, x^4)$

R-R sector $(\sigma, \phi) (x^{\mu}, x^4)$

We consider the R-R zero mode

(Other modes contribute to the boundary state in the same way to the bosonic sector.)

The state

$s_0 = \left( \vartheta^a, \vartheta^a \right)$

satisfies the boundary condition for a Dp-brane.

Then the $D/2D$ bound state is

$|D/2D\rangle = \left( N \vartheta^a \vartheta^a \right)$

T-duality transformation is represented by an operator:

$\Phi \rightarrow \Phi^{*}$

Hori formula

$I_{T\Delta}$ can be written in terms of differential forms as

$I_{T\Delta} = \int_{T\Delta} \left( \mathcal{L}_{1/2} \mathcal{L}_{1/2}^* - \mathcal{L}_{1/2} \mathcal{L}_{1/2}^* \right)

Boundary state on the torus

Boundary state associated with the boundary condition of the open string

T-duality

ex) in case of Dirichlet condition

Calculate the boundary state with the flux associated with $(U(k), C_2 = k)$ on $T^2$

The initial, final state of the closed string associated with the boundary condition of the open string

$U(1)$ in the coordinate $x^4$

Summary and discussion

- Nahm transformation extended nicely to 2d.
- It consistent with T-duality in string theory.
- It indicates $Z_{2}$ duality nature of T-duality
- We introduced the T-duality operator which act both on the boundary state and the RR $\phi$-form state.
- We clarified the relationship between T-duality rule at the superstring level and that at the low energy effective theory