Dynamic Control of Quantum Bits to Maintain Coherence

technische universität
dortmund

Götz S. Uhrig

sabbatical address till March 29: University of NSW, Kensington

5 March 2009, Sendai

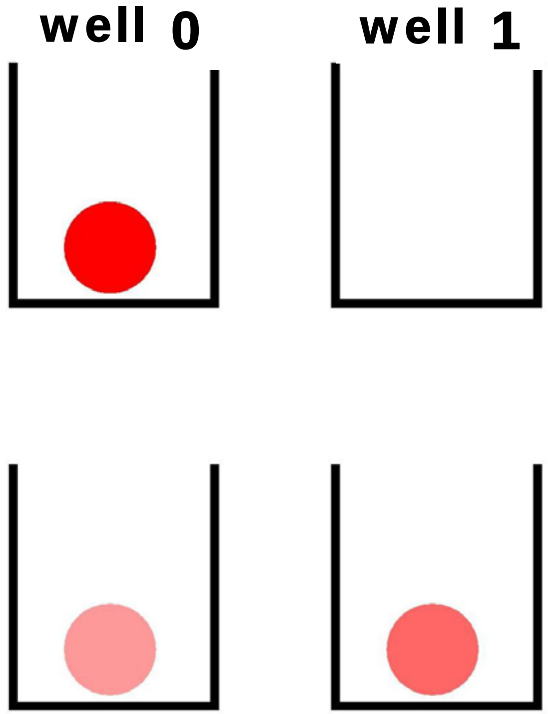
Collaborators:

- Stefano Pasini
- Peter Karbach
- Tim Fischer

Outline

- Introduction: Quantum Bits, Decoherence, and Dynamic Decoupling
- Optimization of Pulse Sequences: Derivation and Proof of Universality
- Effect of the UV Cutoff
- Experimental Verification
- Summary

Why Quantum Bits ?



bit

classical

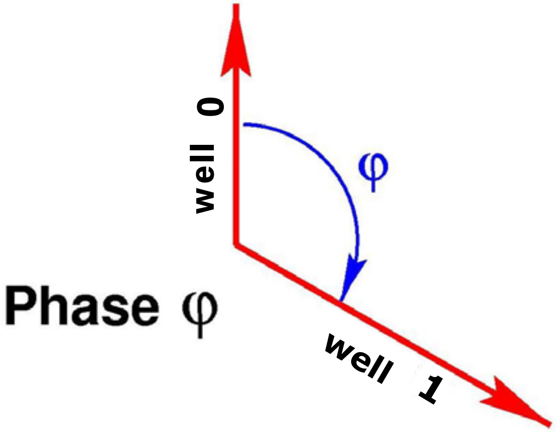
$$|0\rangle \quad \text{or} \quad |1\rangle$$

qubit

quantum mechanical

superpositions possible !

$$|0\rangle + \exp(i\varphi)\,|1\rangle$$

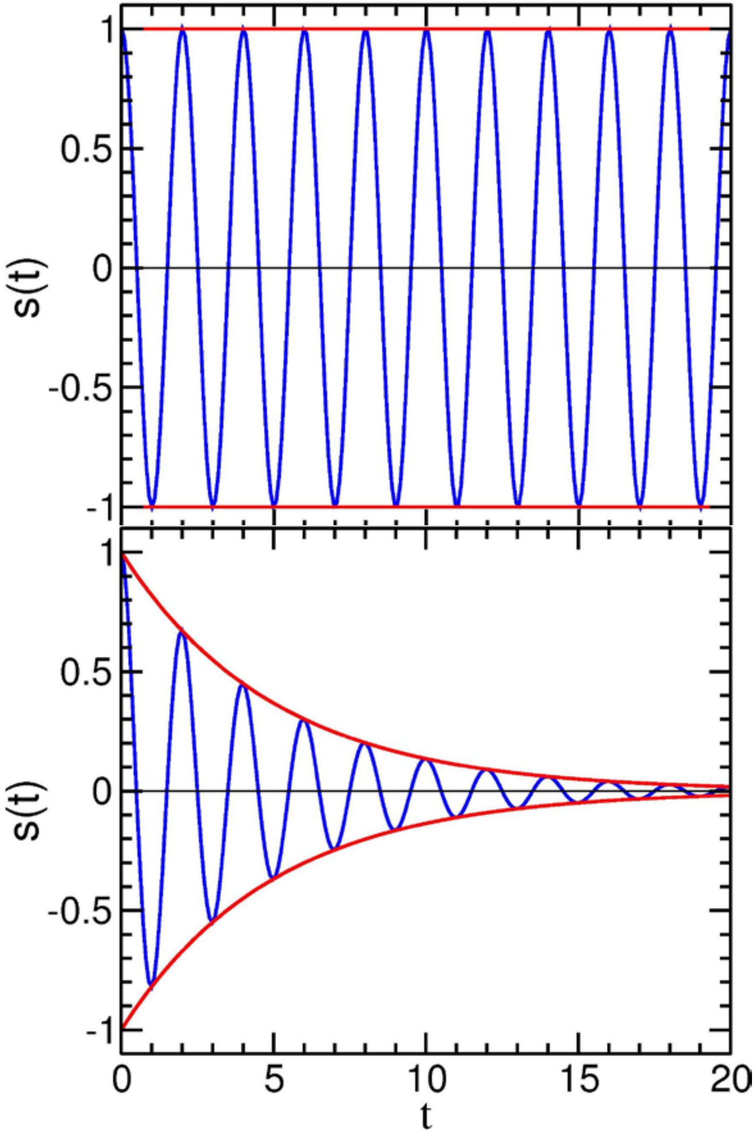


- more information
- but fragile

What is Decoherence ?

coherent
time evolution

decoherent
time evolution



What is Decoherence ?

$$|0\rangle + \exp(\mathrm{i}\varphi)\,|1\rangle \xrightarrow{\text{decoherence}} |0\rangle \text{ or } |1\rangle$$

Example:

$$(|0\rangle|0\rangle + |1\rangle|1\rangle)/\sqrt{2} \xrightarrow{\text{decoherence}} |0\rangle|0\rangle \text{ or } |1\rangle|1\rangle$$

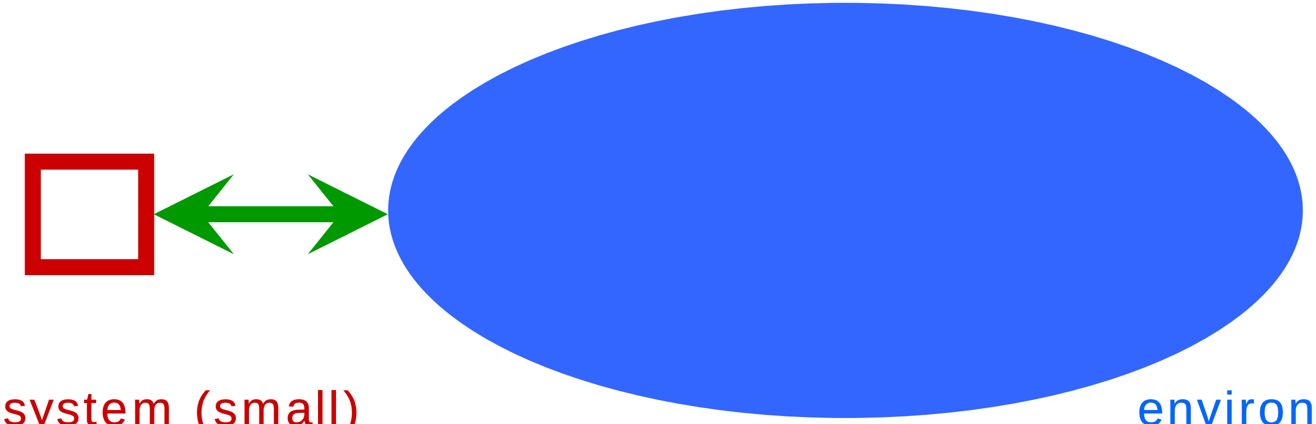
Effect: decoherence destroys all the special properties of quantum information

Introduction: Decoherence

Origin of decoherence ?

- deterministic evolution in time

- interaction with macroscopic environment

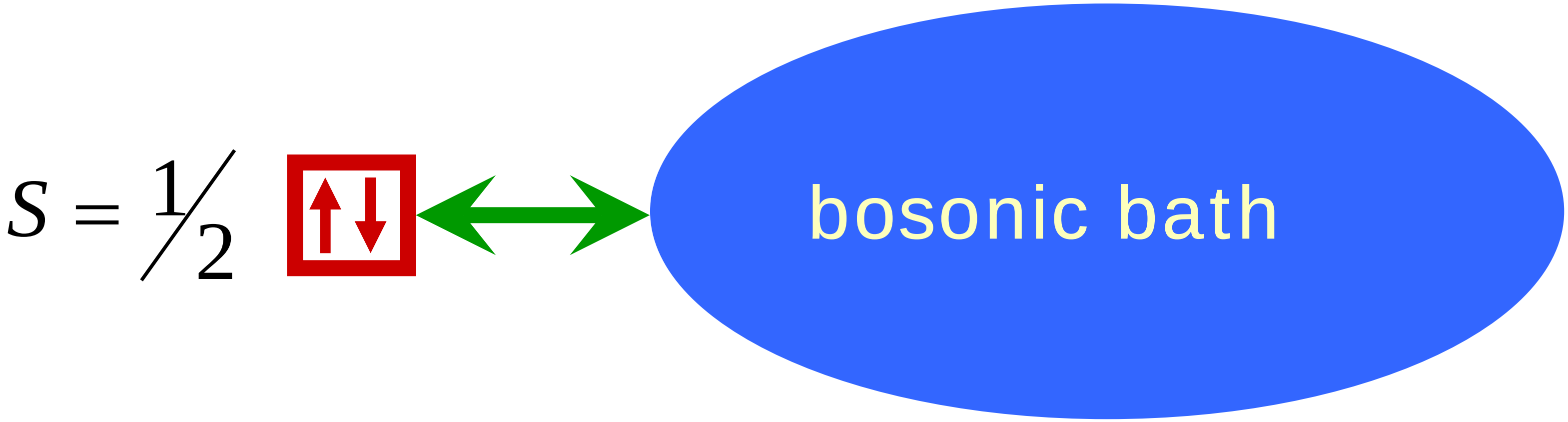


system (small)

environment (huge)

Probabilistic behaviour of the system from tracing out the environment

Phase Decoherence: Analytic Model



$S = \frac{1}{2}$

bosonic bath

Hamiltonian operator

$$H = \underbrace{\Delta\sigma_z}_{\text{Spin/qubit}} + \underbrace{\sum_i \omega_i b_i^+ b_i}_{\text{bath}} + \underbrace{\frac{1}{2}\sigma_z \sum_i \lambda_i (b_i^+ + b_i)}_{\text{coupling}} + E$$

Phase Decoherence: Analytic Model

Hamiltonian

$$H = \Delta\sigma_z + \sum_i \omega_i b_i^+ b_i + \frac{1}{2}\sigma_z \sum_i \lambda_i (b_i^+ + b_i) + E$$

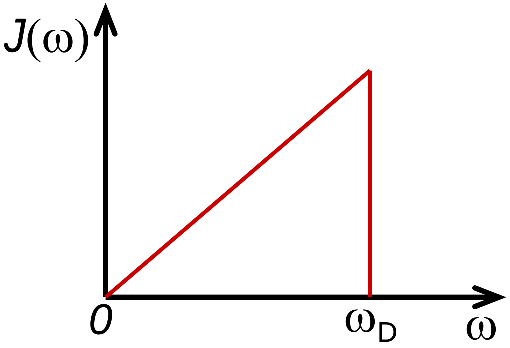
(Leggett et al, RMP`87;
Weiss, World Scientific)

essential information: spectral density

$$J(\omega) = \sum_i |\lambda_i|^2 \delta(\omega - \omega_i)$$

specifically:
ohmic bath

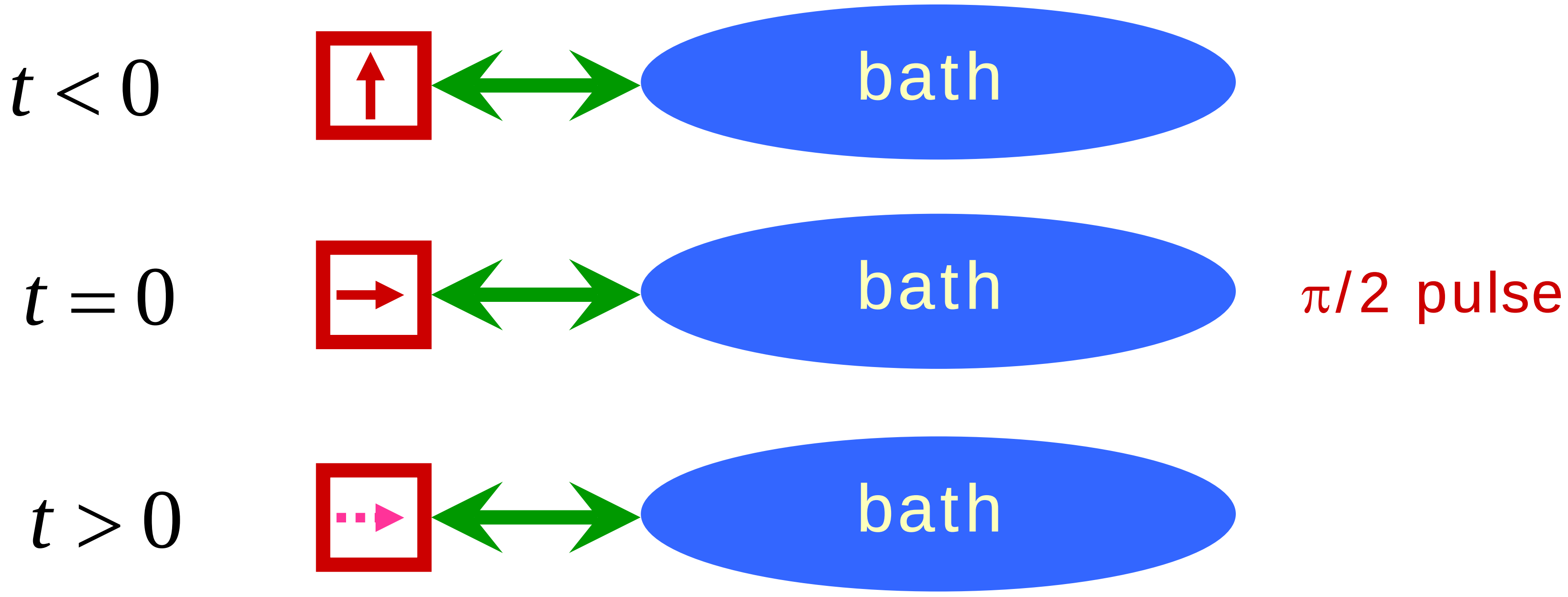
$$J(\omega) = 2\alpha\omega\theta(\omega_D - \omega)$$



possible origins:

- phonons (ω_D Debye frequency)
- photons in cavities
- fermionic particle-hole pairs

Phase Decoherence: Gedanken experiment



$t < 0$ bath

$t = 0$ bath π/2 pulse

$t > 0$ bath

Analytically:

$$
\begin{aligned}
s(t) &= \cos(2\varphi(t))\exp(-2\chi(t)) \quad \text{with} \\
\varphi(t) &= \frac{1}{2}\int_0^\infty J(\omega)\frac{\sin(\omega t)}{\omega^2}d\omega \\
\chi(t) &= \int_0^\infty J(\omega)\frac{\sin(\omega t/2)^2}{\omega^2}\coth(\beta\omega/2)d\omega
\end{aligned}
$$

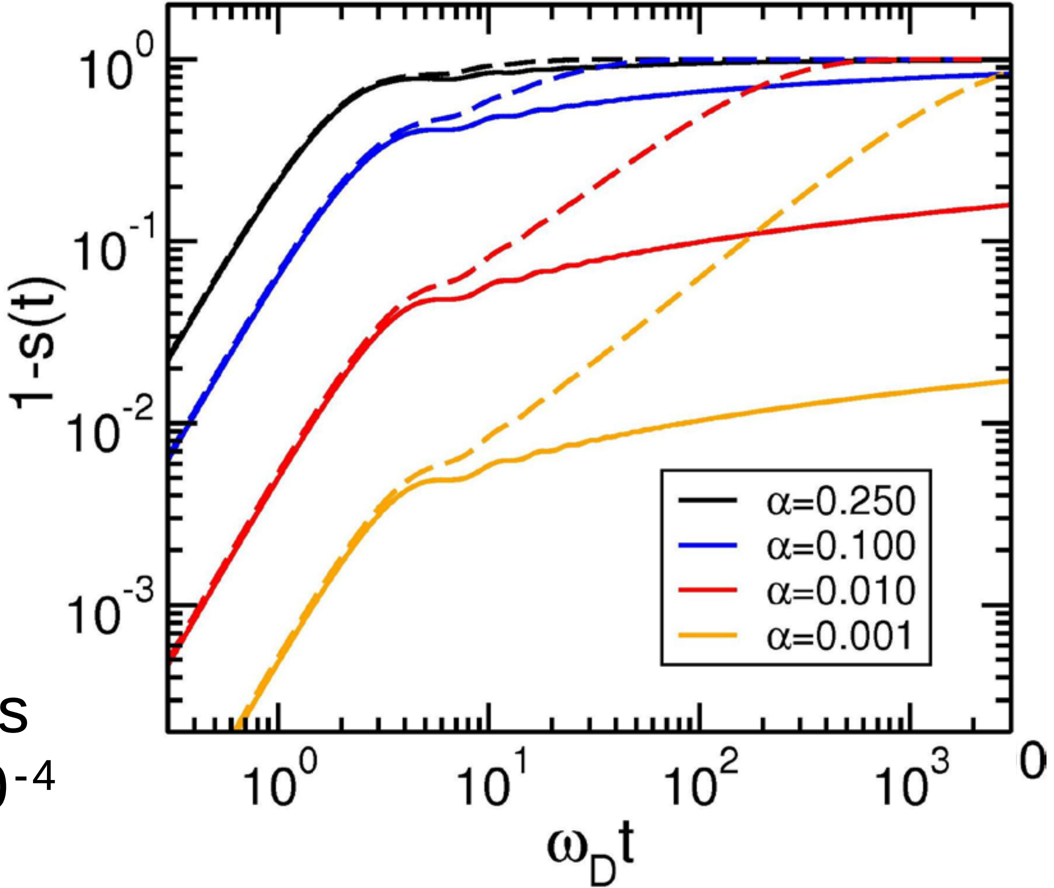
Phase Decoherence: Results

$$s(t) = \cos(2\varphi(t))\exp(-2\chi(t)) \quad \text{with}$$

$$\varphi(t) = \frac{1}{2}\int_0^\infty J(\omega)\frac{\sin(\omega t)}{\omega^2}d\omega$$

$$\chi(t) = \int_0^\infty J(\omega)\frac{\sin(\omega t/2)^2}{\omega^2}\coth(\beta\omega/2)d\omega$$

Dashed curves
at finite temperature



Only for very short times
the error stays below 10^{-4}

Means against decoherence ?

Remedies ?

- Insulation (↔ controllability)
- Algorithmic error correction
- Dynamical decoupling

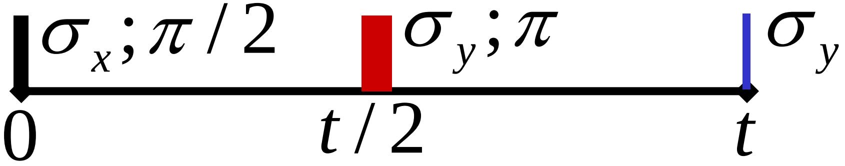
What is “Dynamical Decoupling” ?

Dynamical
Decoupling ?

Static Decoupling: Spin Echo I

(Hahn, PR’50)

pulse sequence:



$$H = h\sigma_z \xrightarrow{\pi\text{ pulse}} H = -h\sigma_z$$

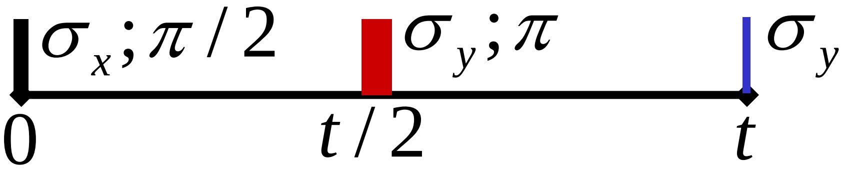
Evolution in time

$$1 = \exp(iHt/2\hbar)\overset{\pi\text{ pulse}}{|}\exp(-iHt/2\hbar)$$

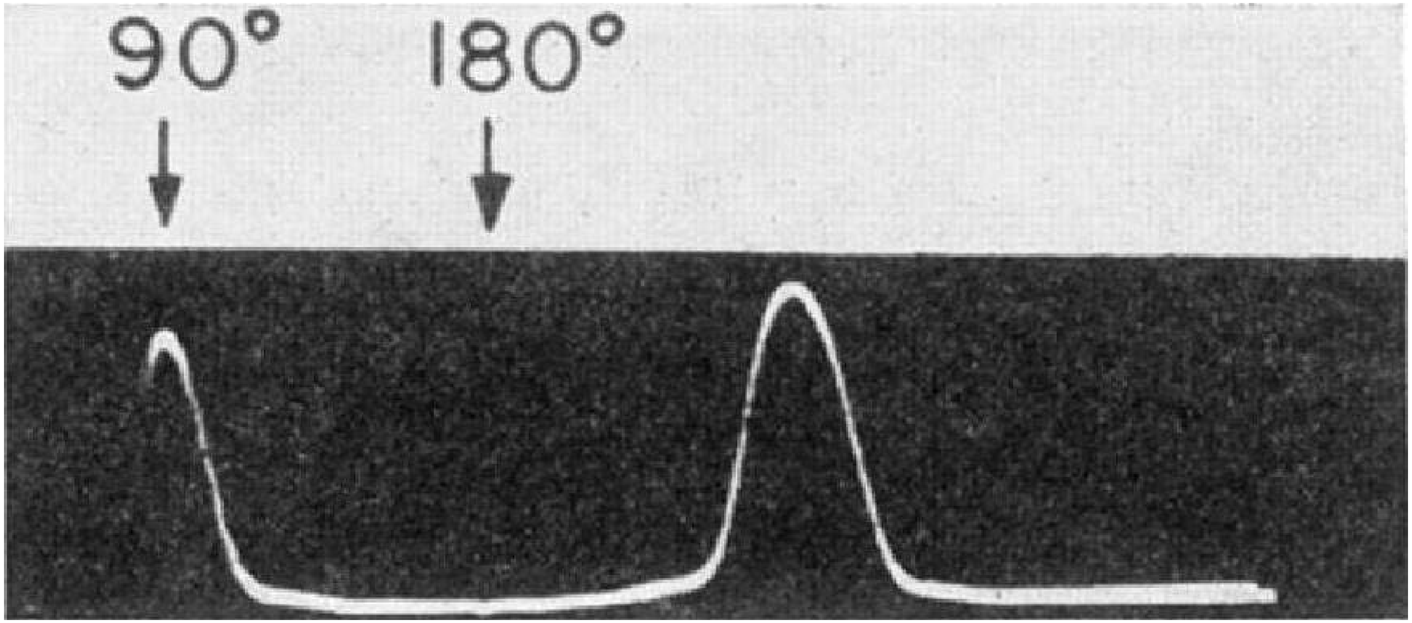
No influence of the coupling !

Static Decoupling: Spin Echo II

pulse sequence:



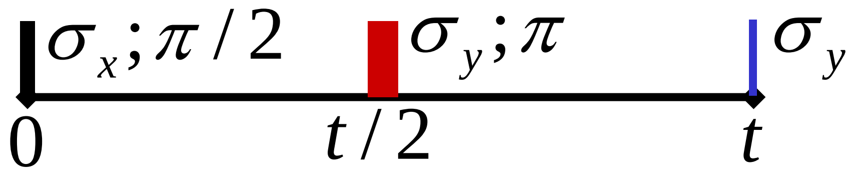
Liquid NMR)



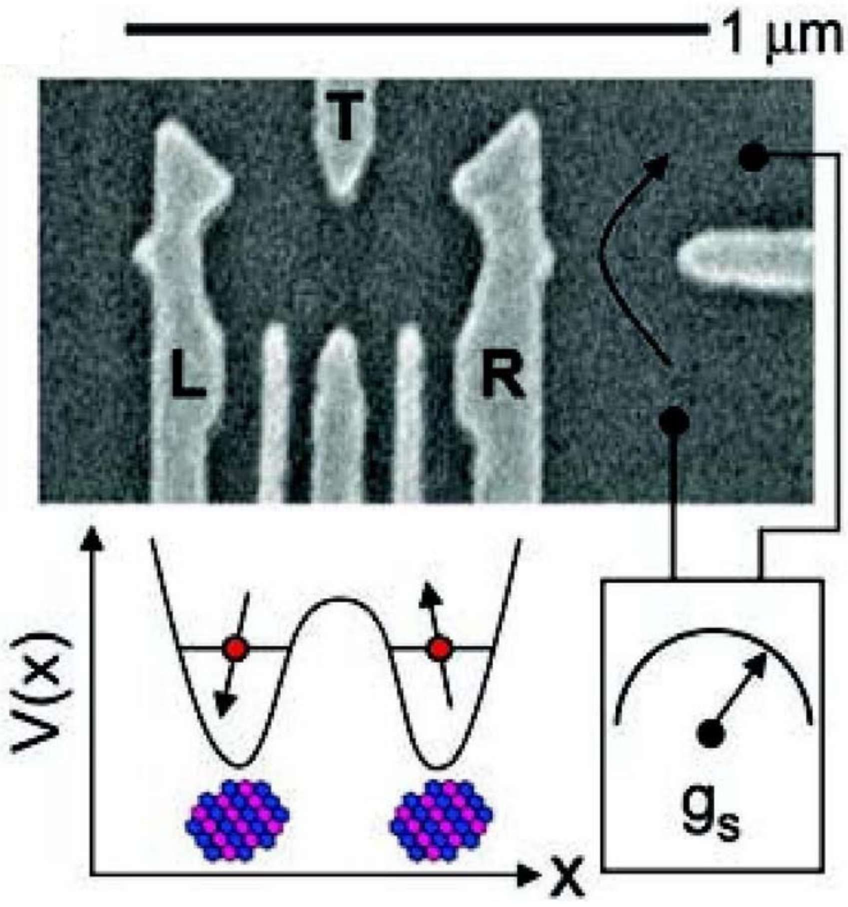
(Carr/Purcell, PR'54)

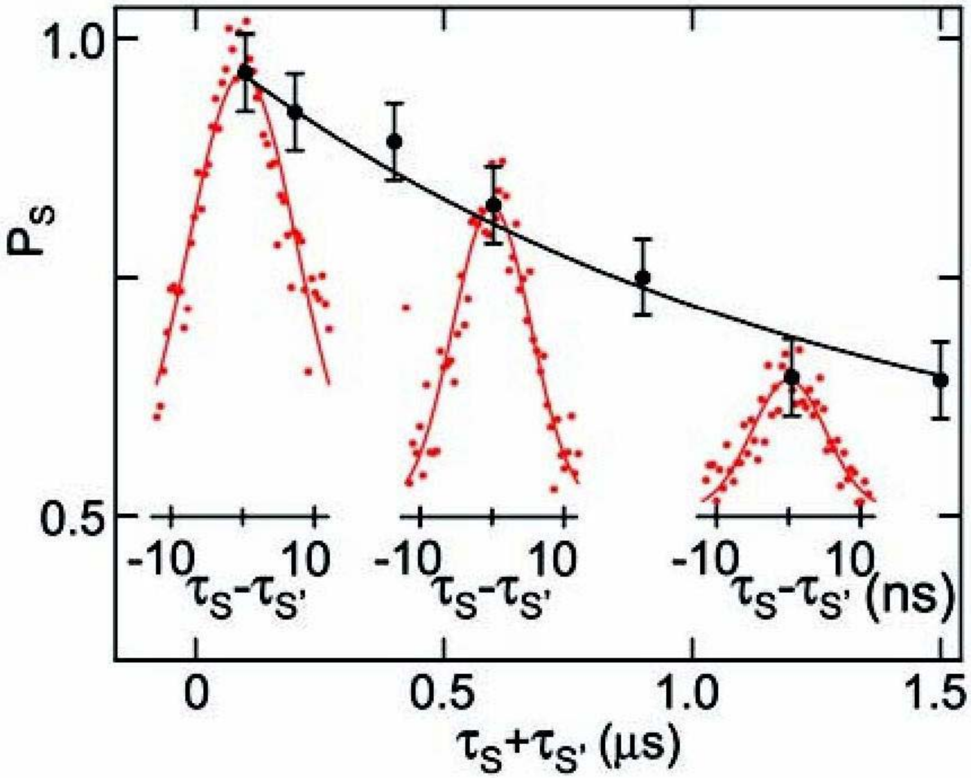
Static Decoupling: Spin Echo III

pulse sequence:



semiconductor quantum dots





(Petta et al., Science’05)

Dynamical Decoupling I

HERE: dynamical (temporal) fluctuations

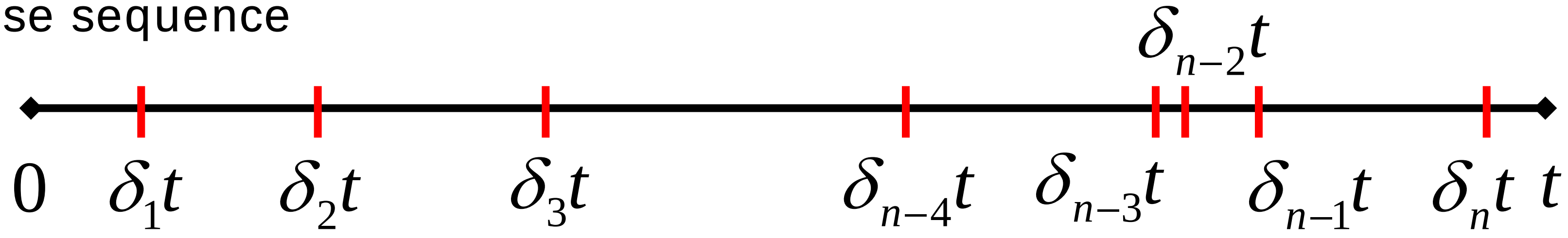
$$H_{\text{coupling}} = \frac{1}{2}\sigma_z \sum_i \lambda_i (b_i^+ + b_i) \approx \frac{1}{2}\sigma_z \sum_i \lambda_i \left\langle b_i^+ + b_i \right\rangle (t)$$

$\left\langle b_i^+ + b_i \right\rangle (t)$ time-dependent magnetic field

- spin echo correction can be even destructive !
- several π pulses necessary: pulse sequences
- time intervals as short as possible

Dynamical Decoupling II

Pulse sequence



General Result

(Uhrig, PRL‘07)

$$\begin{aligned} s_n(t) &= \cos(2\varphi_n(t))\exp(-2\chi_n(t)) \quad \text{with} \\ \varphi_n(t) &= \int_0^\infty \frac{J(\omega)}{2\omega^2} x_n(\omega t) d\omega \\ \chi_n(t) &= \int_0^\infty \frac{J(\omega)}{4\omega^2} \coth(\beta\omega/2) \boxed{|y_n(\omega t)|^2} d\omega \end{aligned}$$

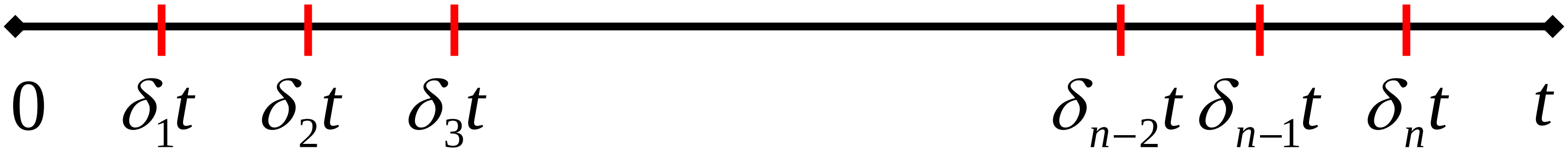
with

$$x_n(z) = (-1)^n \sin(z) + 2\sum_{m=1}^{n} (-1)^{m+1} \sin(z\delta_m)$$

$$\boxed{y_n(z) = 1 + (-1)^{n+1} e^{iz} + 2\sum_{m=1}^{n} (-1)^m e^{iz\delta_m}}$$

Dynamical Decoupling III

So far: equidistant pulse sequences (Viola/Lloyd PRA`98;Ban JMO`98)



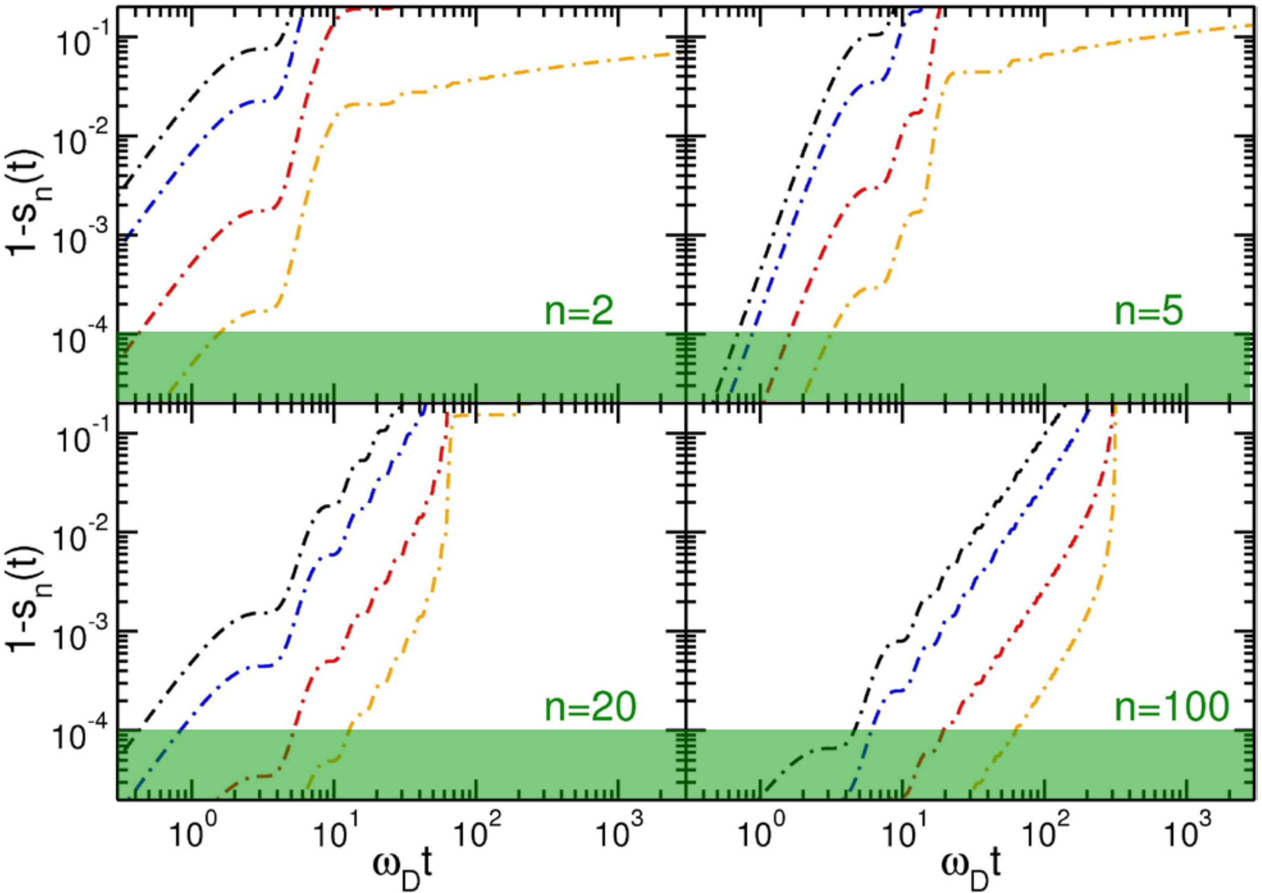
From top
to bottom

$\alpha=0.250$

$\alpha=0.100$

$\alpha=0.010$

$\alpha=0.001$



Optimization of Pulse Sequence

Optimization
of sequence ?

What can be optimized ?

Optimized pulse sequence

(Uhrig, PRL‘07)

Exponent of decoherence $\chi_n(t) = \int_0^\infty \frac{J(\omega)}{4\omega^2} \coth(\beta\omega/2) |y_n(\omega t)|^2 d\omega$

We require:

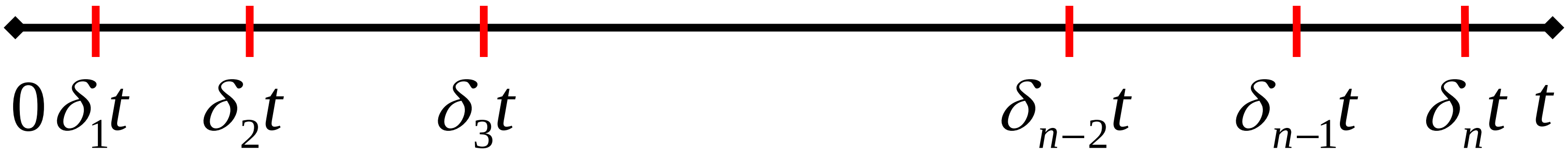
first *n* derivatives vanish

$$y_n(z) = 1 + (-1)^{n+1} e^{iz} + 2 \sum_{m=1}^{n} (-1)^m e^{iz\delta_m}$$

$$\Rightarrow \quad \boxed{\delta_j = \sin^2\left({j\pi}\big/{2(n+1)} \right)}$$

Optimized Dynamic Decoupling I

Optimized pulse sequence



Solid curves are optimized

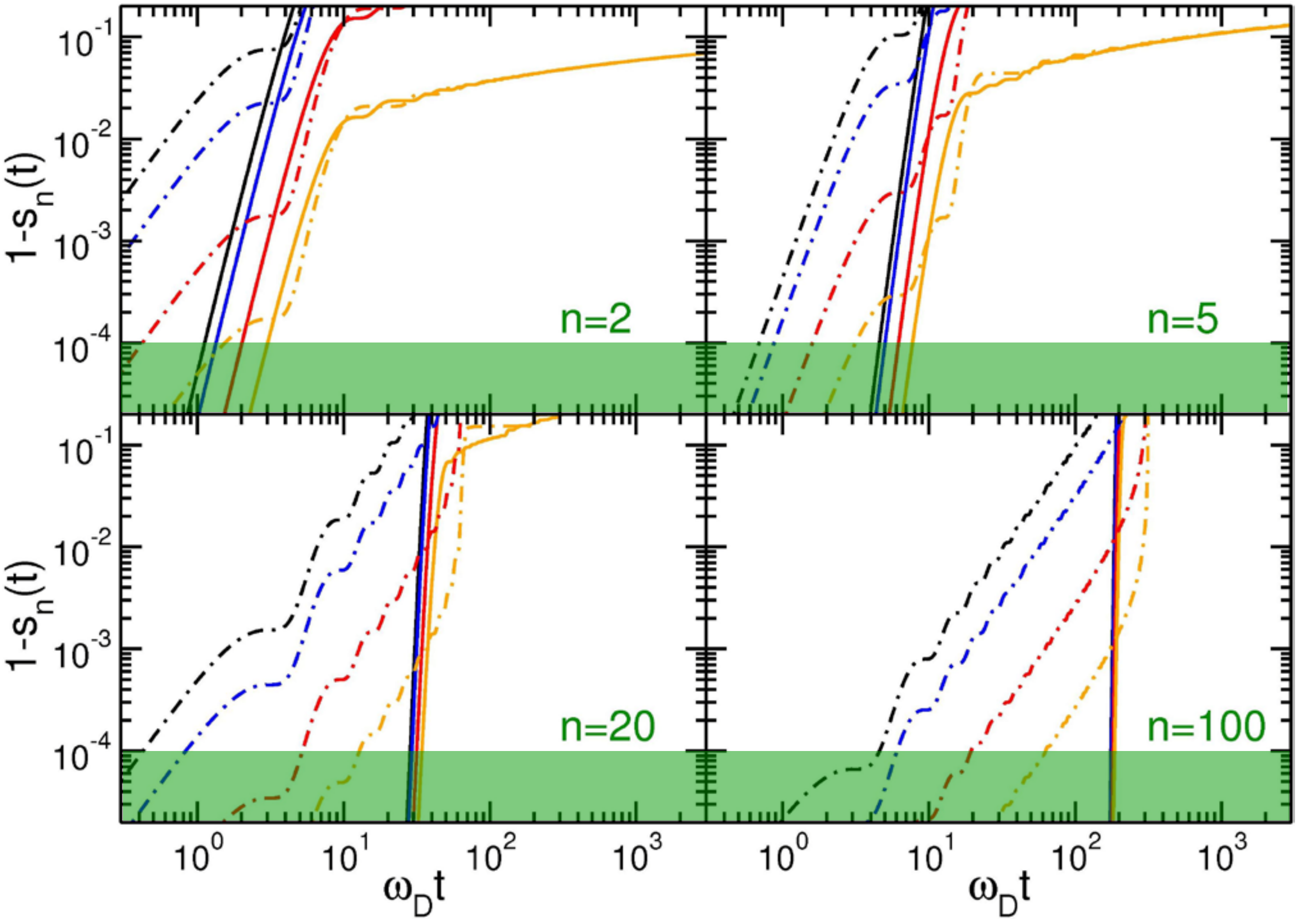
From top to bottom

$\alpha=0.250$

$\alpha=0.100$

$\alpha=0.010$

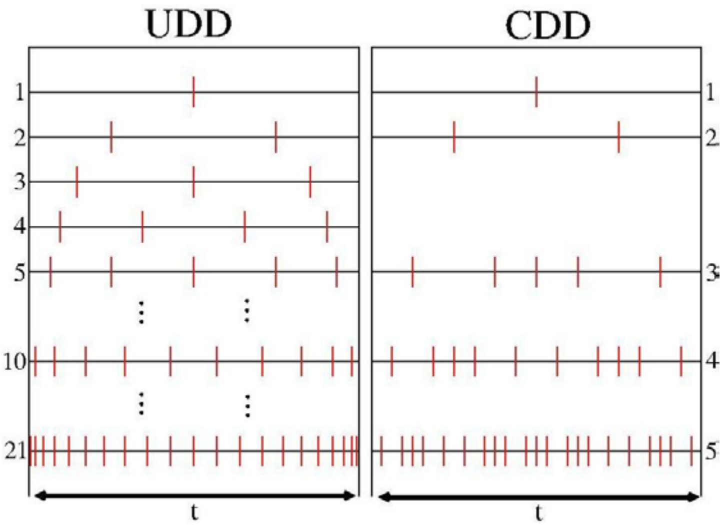
$\alpha=0.001$



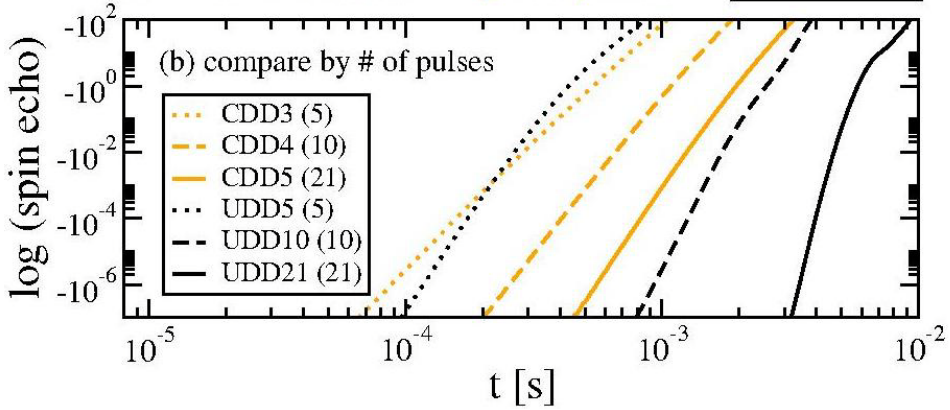
Optimized Dynamic Decoupling III

Optimized sequence

Concatenated sequence



GaAs quantum dot: Bath of nuclear spins



(Lee,Witzel,DasSarma, PRL‘08)

UDD better than CDD

Claim:

UDD generally optimum, independent from bath

(Lee, Witzel,DasSarma, PRL’08; Uhrig, NJP´08)

Dynamic Decoupling: General Dephasing I

General T_2 dephasing Hamiltonian

$$H = \sigma_z A_1 + A_0$$

$$\widetilde{R}_{\uparrow\downarrow}(t') = \sum_{j=0}^{\infty} (-i)^j \sum_{\underline{m} \in B_j} C_{\underline{m}}^{\uparrow\downarrow}(t') A_{m_j} A_{m_{j-1}} \cdots A_{m_2} A_{m_1}$$

$C_{\underline{m}}^{\uparrow\downarrow}(t')$ vanish for all odd numbers of A_1 !

- ➢ analytically for all orders $n \leq 9$ (Lee/Witzel/Das Sarma PRL`08)
- ➢ analytically for all orders $n \leq 14$ (Uhrig NJP`08)

To be presumed: General applicability of optimized sequence !

Dynamic Decoupling: General Dephasing II

General T_2 dephasing Hamiltonian

$$H = \sigma_z A_1 + A_0$$

$$\exp(iA_0 t) A_1 \exp(-iA_0 t) = \sum_{p=0}^{\infty} \hat{Z}_p t^p$$

Expansion in total evolution time T implies one has to show the vanishing of

$$\hat{\Delta}_n = \sum_{\{p_j\}} [\hat{Z}_{p_n} \cdots \hat{Z}_{p_2} \hat{Z}_{p_1} F_{p_1, p_2, \ldots, p_n} T^{n+p_1+p_2 \cdots + p_n}],$$

with $$F_{p_1,\ldots,p_n} \equiv \int_0^T \frac{dt_n}{T} \cdots \int_0^{t_3} \frac{dt_2}{T} \int_0^{t_2} \frac{dt_1}{T} \prod_{j=1}^{n} F_N(t_j) \left(\frac{t_j}{T}\right)^{p_j}$$

Their vanishing is proven via the recursion of

$$\int_0^{\pi} d\theta_n \cdots \int_0^{\theta_3} d\theta_2 \int_0^{\theta_2} d\theta_1 \prod_{j=1}^{n} \cos(r_j \theta_j + q_j \theta_j) = 0$$ for n odd

for n being odd, r_j being an odd multiple of $(N+1)$, $\sum_{j=1}^{n} |q_j| \leq N$

This can be shown by successive integration where $r_j \rightarrow R_j$ and $q_j \rightarrow Q_j$, with the same properties

Qed !

(Yang/Liu PRL`08)

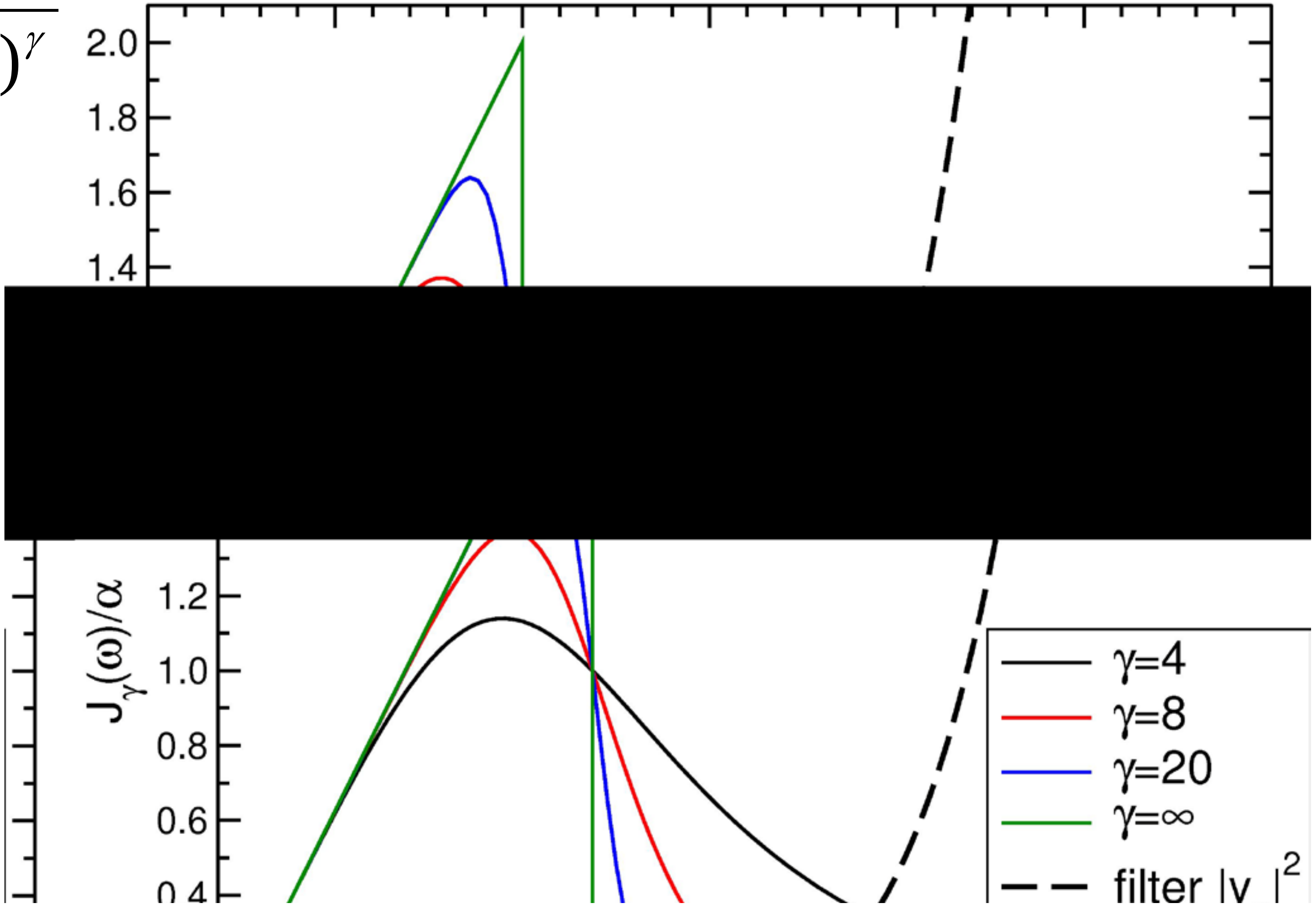
Effect of the UV-Cutoff

Effect of the
Ultraviolet- Cutoff ?

Effect of the UV-Cutoff I

Spin-boson model as before, now with spectral density:

$$J_\gamma(\omega) = \frac{2\alpha\omega}{1+(\omega/\omega_D)^\gamma}$$



[redacted]

(Uhrig NJP`08)

Effect of the UV-Cutoff II

$$J_\gamma(\omega) = \frac{2\alpha\omega}{1+(\omega/\omega_D)^\gamma}$$

BB: bang-bang control

(Viola/Lloyd PRA`98; Ban JMO`98)

CDD:
concatenated dynamic decoupling

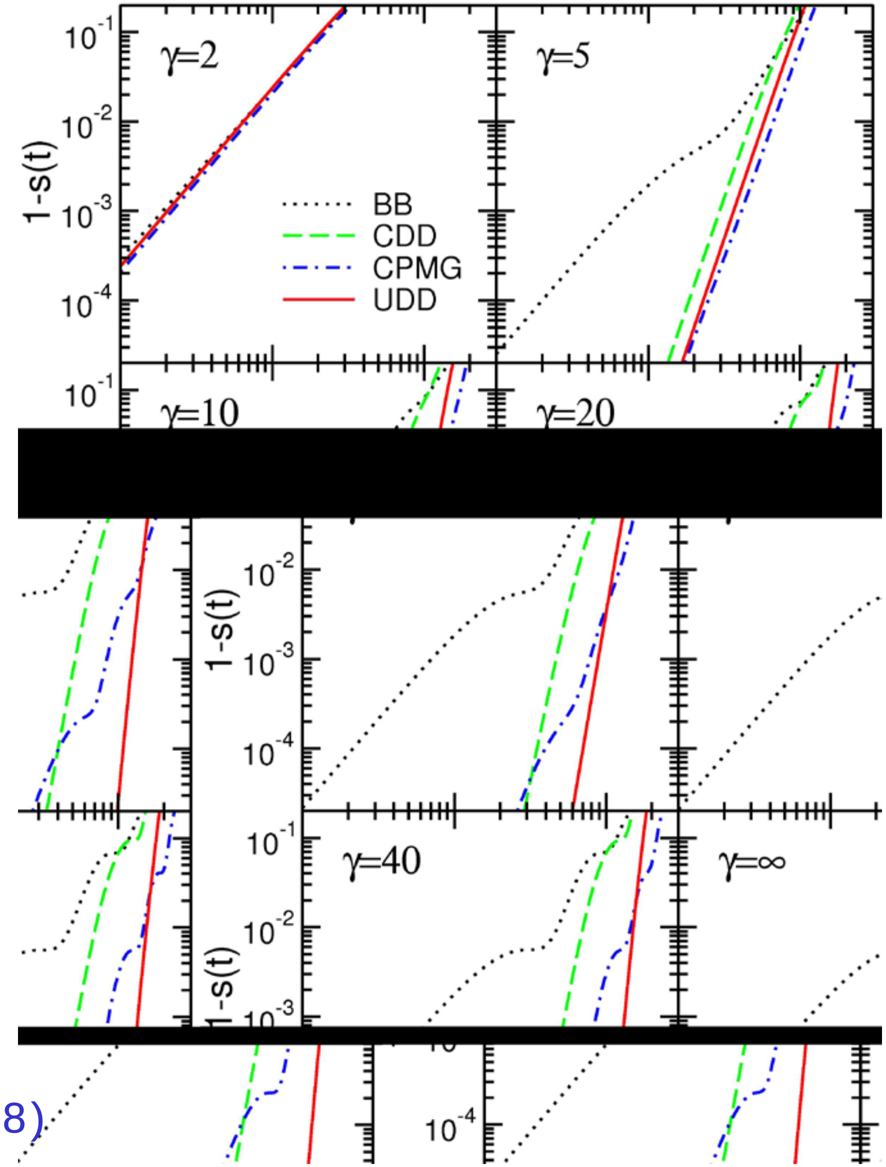
(Khodjasteh/Lidar PRL `05)

CPMG:
Carr-Purcell-Meiboom-Gill

(Carr/Purcell, PR'54; Meiboom/Gill RSI´58)

UDD:
optimized pulse sequence

(Uhrig PRL`07)



(Uhrig NJP`08)

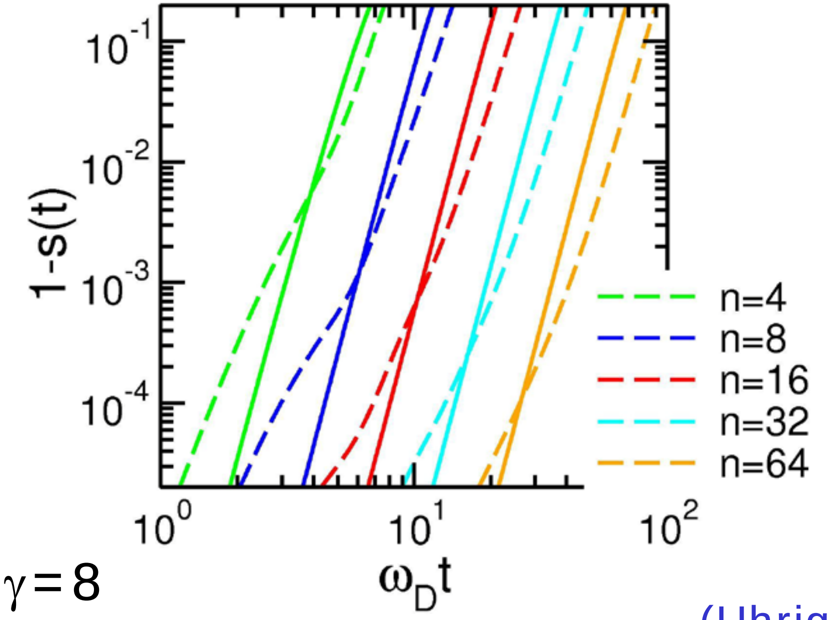
Effect of the UV-Cutoff II

Message:

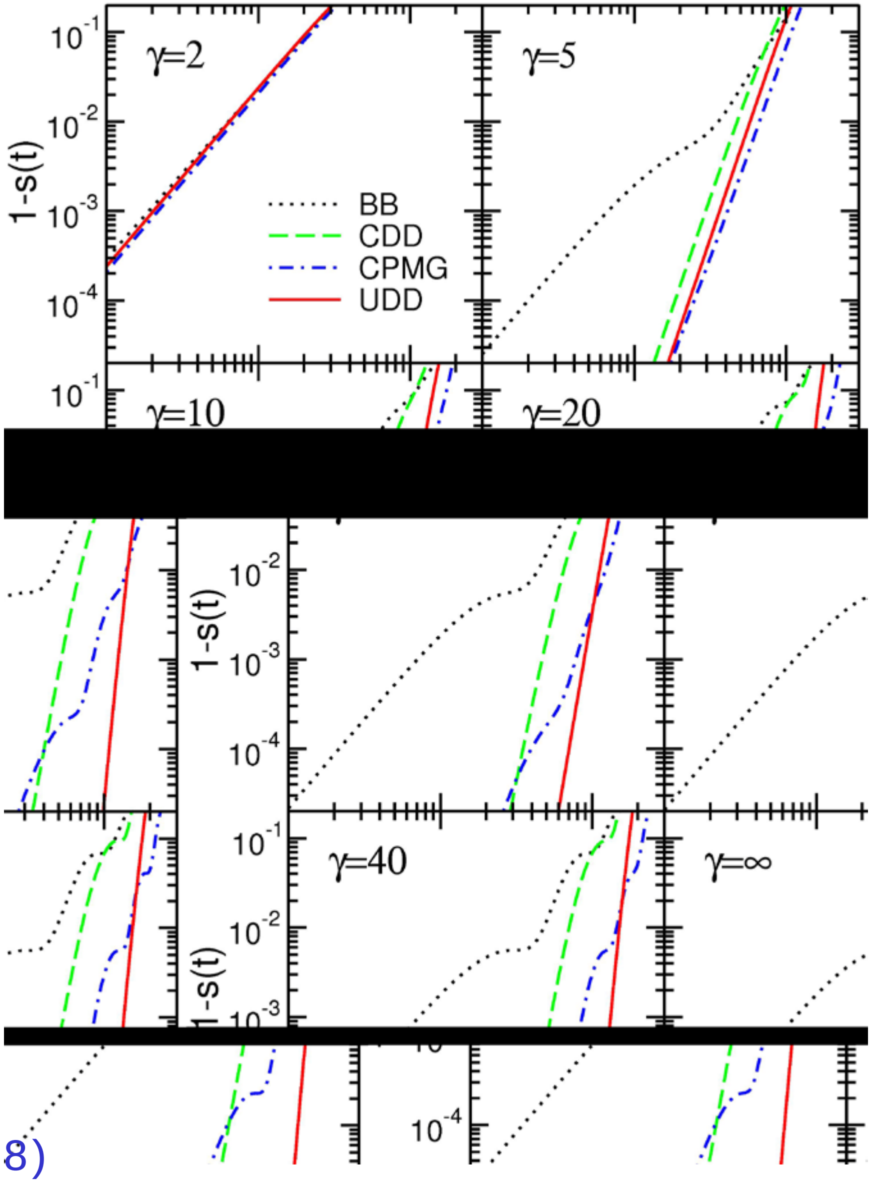
CPMG and UDD are the most competitive in this model

CPMG for soft cutoffs

UDD for hard cutoffs



$\gamma=8$



[redacted]

[redacted]

(Uhrig NJP`08)

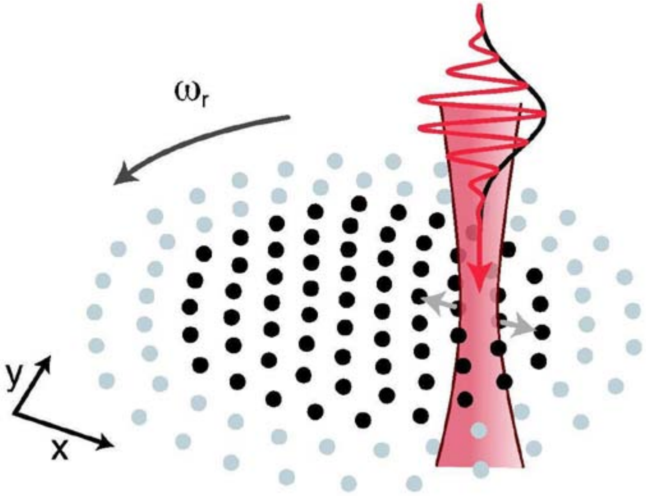
Experimental Verification

Experimentally realizable and verifiable ?

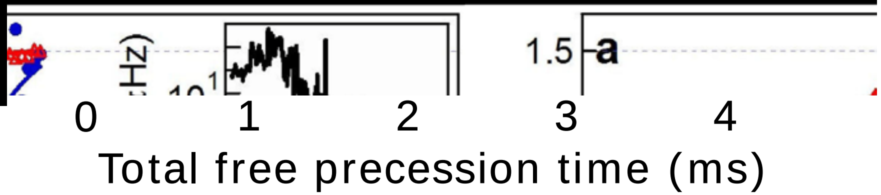
Experimental Verification I

Work by H. Uys, M.J. Biercuk et al.
in the group of J.J. Bollinger, NIST Boulder
Jan. 2009

- About 10000 Be^9 ions in a Penning trap
- Form a Wigner crystal
- Optically (laser) induced spin flip transitions



Experimental Verification: Results



Ambient noise: very soft cutoff

$$J(\omega) \propto 1/\omega^{4}$$

$\Rightarrow$ CPMG better than UDD

Simulated ohmic noise: hard cutoff

$$J(\omega) \propto \omega\,\Theta(\omega_{\mathrm{D}} - \omega)$$

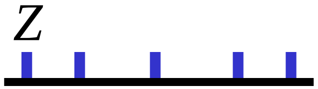
$\Rightarrow$ UDD better than CPMG

Extension of T_2 and T_1

Can spin flips also be suppressed ?

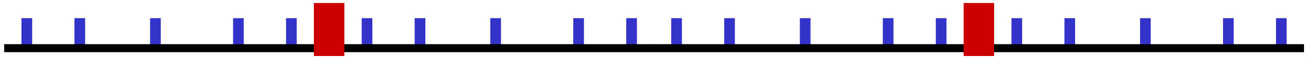
Concatenated UDD Sequences

$p^{m}{}_{\mathrm{UDD}}$ Optimized sequence of m pulses, suppressing spin flips up to T^{m+1}



p_{CPMG} Built from $p^{m}{}_{\mathrm{UDD}}$, suppressing dephasing up to T^3

$$p_{\mathrm{CPMG}} = p^{m}{}_{\mathrm{UDD}} X \; p^{m}{}_{\mathrm{UDD}} \; p^{m}{}_{\mathrm{UDD}} X \; p^{m}{}_{\mathrm{UDD}}$$



Iterated concatenation according to $p_{\mathrm{n+1}} = p_n X \; p_n X$

makes arbitrary suppression of spin flips possible !

(Uhrig, arXiv:0810.5616, PRL in press)

Summary

Almost done !

Summary

- Basic model for dephasing decoherence
- Optimized pulse sequence (UDD) $\boxed{\delta_j = \sin^2\left({j\pi}/{2(n+1)} \right)}$
- Importance of a hard UV Cutoff
- Experimental verification
- Tractability of general decoherence (CUDD)

Optimized Dynamic Decoupling II

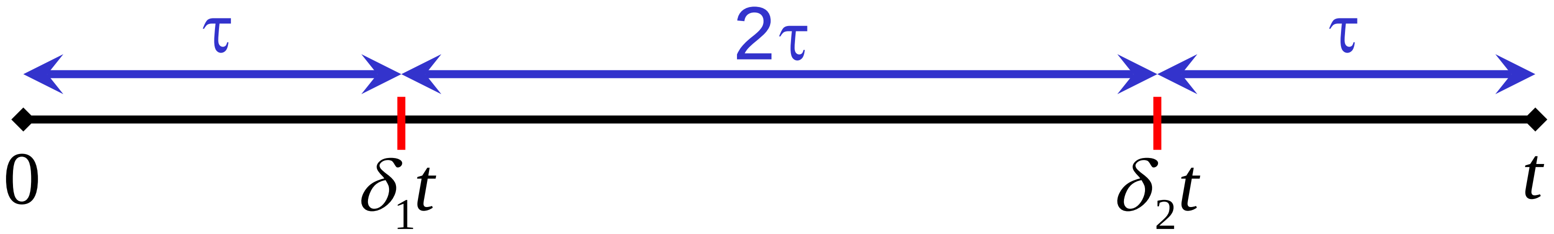
Optimized pulse sequence:

Context to previously known pulse sequences

$$\boxed{\delta_j = \sin^2\left({}^{j\pi}\!/_{2(n+1)} \right)}$$

For $n=2$: $\delta_1=1/4$ and $\delta_2=3/4$

Reproduces the well-known Carr-Purcell-Meiboom-Gill (CPMG) cycle!



Other recent investigations of pulse sequences:

Cappellaro et al. JCP`06; Witzel/Das Sarma PRL`07; Khodjasteh/Lidar PRL`05; Viola/Knill PRL`05; Yao/Liu/Sham PRL`07; Möttönen et al. PRA`06; ...

Dynamic Decoupling: Operator Level

Relevant

(Uhrig, NJP`08)

Not special experiment, BUT time evolution operator

$\boxed{\tilde{R}_\sigma(t)}$ may not depend on spin $\boxed{\sigma}$

$$\begin{aligned} \Delta(t) &:= \tilde{R}_\uparrow - \tilde{R}_\downarrow \approx 0 \\ &= e^{-iH^{\mathrm{eff}}t} \, e^{-i\phi_n(t)} \left[e^{\Delta_n K} - e^{-\Delta_n K}\right] \end{aligned}$$

with

$$\Delta_n K := \sum_i \frac{\lambda_i}{2\omega_i}\left(b_i^\dagger \; y_n(\omega_i t) - b_i \; y_n^*(\omega_i t)\right)$$

hence

$$y_n(z) = \mathcal{O}(z^{n+1}) \quad \Leftrightarrow \quad \Delta(t) = \mathcal{O}(t^{n+1})$$

Effect of the UV- Cutoff IV

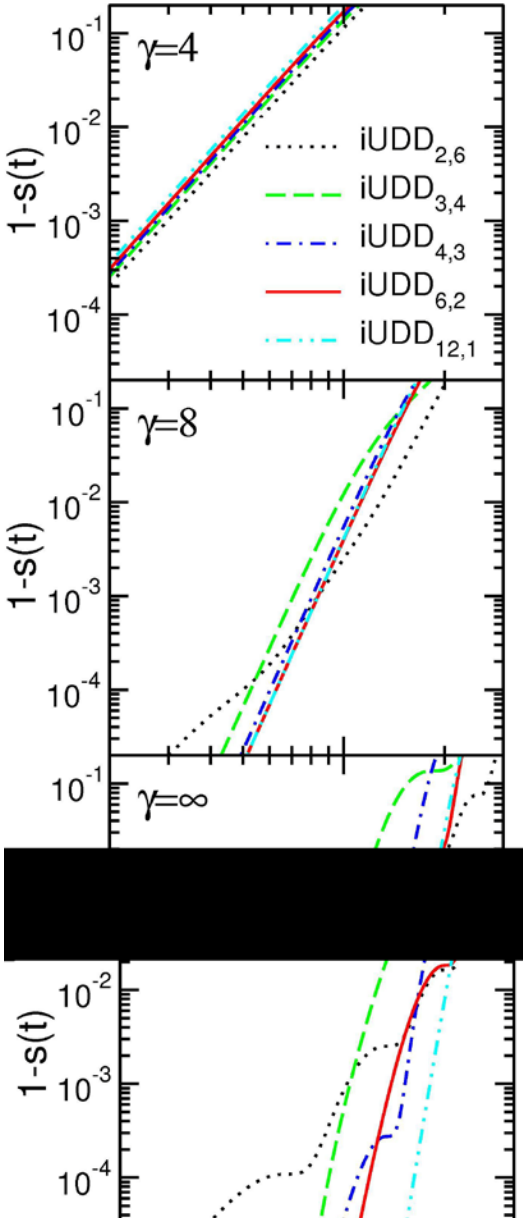
Tradeoff possible
depending on γ:

Iterated UDD sequences

iUDD$_{m,c}$

m is # of pulses in one cycle ($m=2$ is CPMG)

c is # of cycles



[redacted]

(Uhrig NJP`08)