Key symmetries of superconductivity

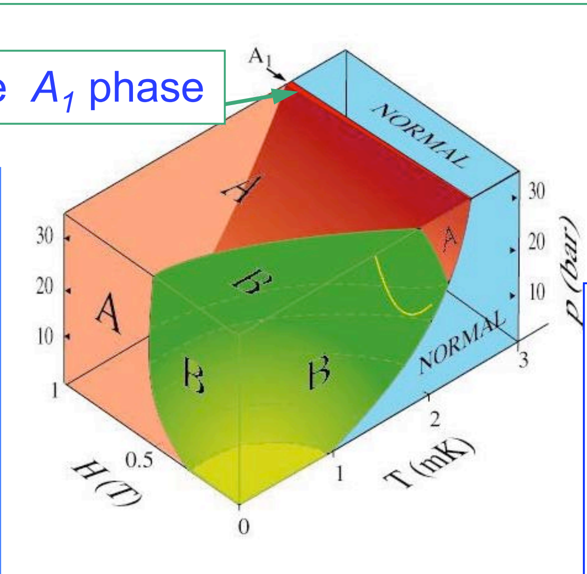
Inversion and time reversal symmetry

Sendai, March 2009

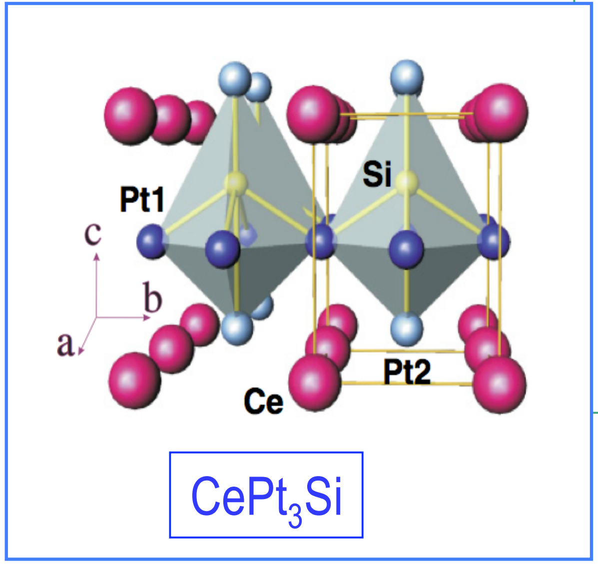
Manfred Sigrist, ETH Zürich

1st GCOE

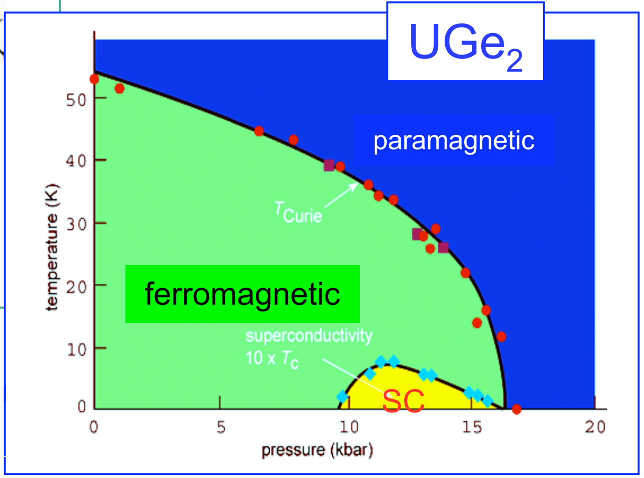
International Symposium



^{3}He A_1 phase



$CePt_3Si$



UGe_2

Key symmetries of superconductivity

Inversion and time reversal symmetry

Sendai, March 2009

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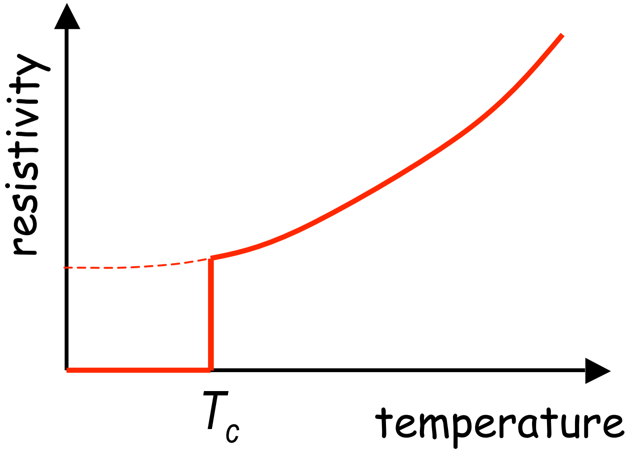
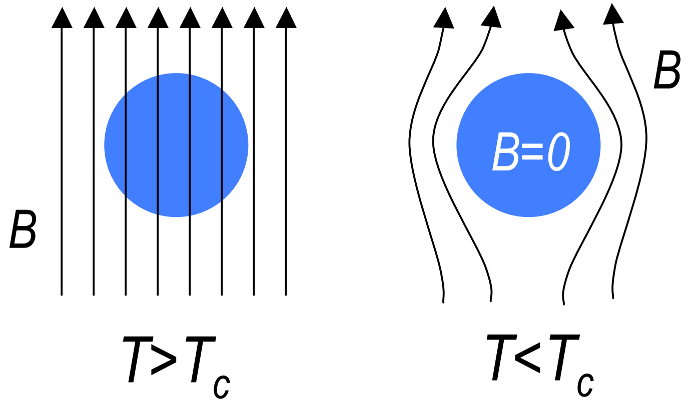
- *superconductivity:* general introduction
- *Cooper pairing:* symmetry aspects
  role of inversion and time reversal symmetry
- superconductivity in the absence of inversion and time reversal sym.
- lack of inversion symmetry: non-centrosymmetric superconductors
  and some of their physical properties

Superconductivity

Electrical resistance (1911)



Field expulsion (1933)
Meissner-Ochsenfeld effect



Superconductivity as a thermodynamic phase

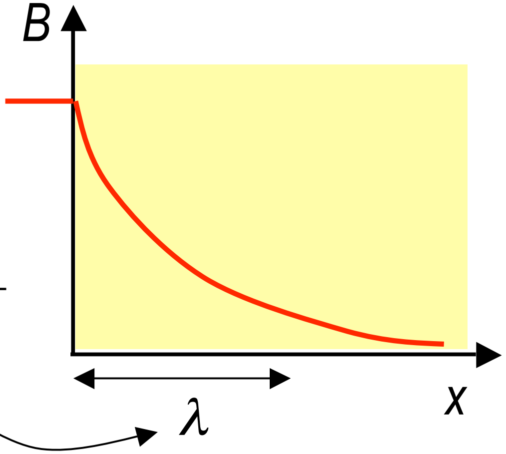
London theory (1935)

$$\left.\begin{aligned} \nabla\times\lambda^2\vec{j} &= -\vec{B} \\ \nabla\times\vec{B} &= \frac{4\pi}{c}\vec{j} \end{aligned}\right\} \Rightarrow \boxed{\nabla^2\vec{B}=\lambda^{-2}\vec{B}}$$

density of superconducting electrons

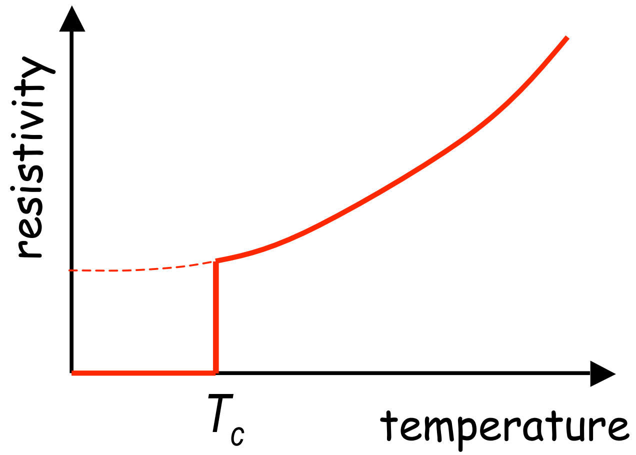
$$\lambda^{-2}=\frac{4\pi e^2 n_s}{mc^2}$$

London penetration depth



Superconductivity

Electrical resistance (1911)



Field expulsion (1933)
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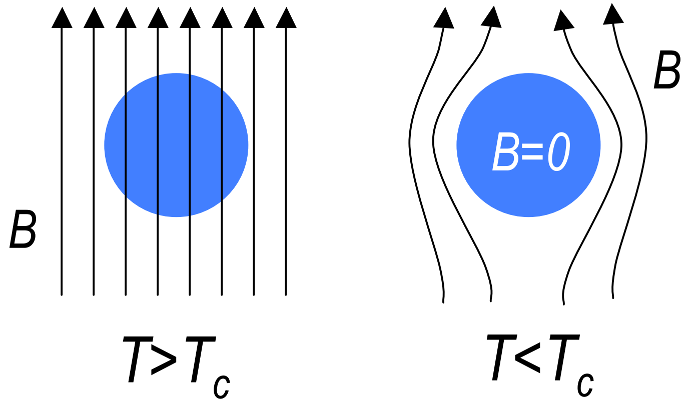


Superconductivity as a thermodynamic phase

Ginzburg-Landau theory (1950)

Superconductivity described by
a complex macroscopic wave function

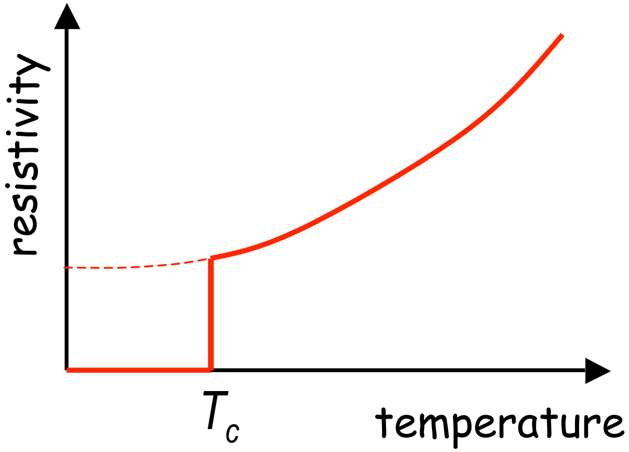
order parameter

$$\Psi(\vec{r}) = |\Psi(\vec{r})|e^{i\phi(\vec{r})}$$

→ Phenomenology of superconductivity

Superconductivity

Electrical resistance (1911)



Field expulsion (1933)
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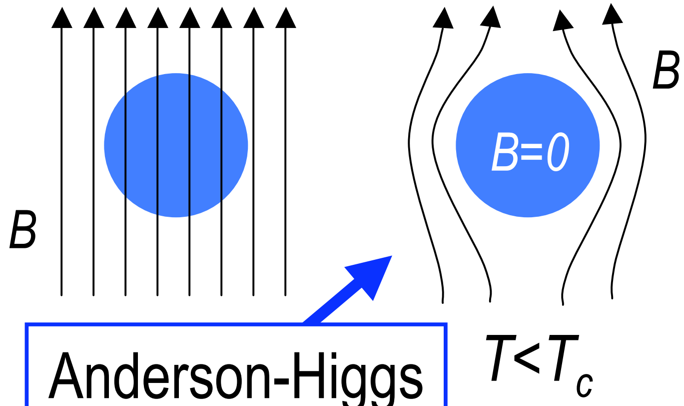


Anderson-Higgs mechanism

S

C

$\Psi(\vec{r})$ violates *U(1)*-gauge symmetry

superconductivity

order parameter

$$\Psi(\vec{r}) = |\Psi(\vec{r})| e^{i\phi(\vec{r})}$$

Phenomenological point of view

Spontaneous symmetry breaking

supplement

Ginzburg-Landau theory

2nd order phase transition from normal to superconducting state

spontaneous symmetry breaking

order parameter: $$\boxed{\Psi = |\Psi| e^{i\phi}}$$ *macroscopic wave function*

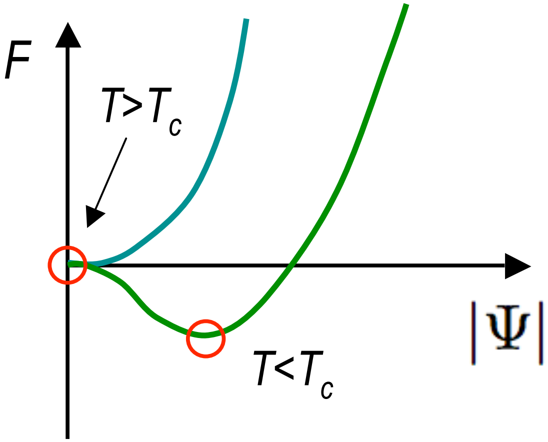
$|\Psi| = 0$ normal phase $T > T_c$

$|\Psi| \neq 0$ superconducting phase $T < T_c$

Free energy expansion at T_c

$$F[\Psi] = a(T - T_c)|\Psi|^2 + b|\Psi|^4$$

$a, b > 0$ scalar under *U(1)*-gauge operation $\Psi \rightarrow \Psi e^{i\chi}$



supplement

Ginzburg-Landau theory

2nd order phase transition from normal to superconducting state

spontaneous symmetry breaking

order parameter: $$\boxed{\Psi = |\Psi| e^{i\phi}}$$

macroscopic
wave function

$$|\Psi| = \frac{a(T_c - T)}{2b}$$ determined

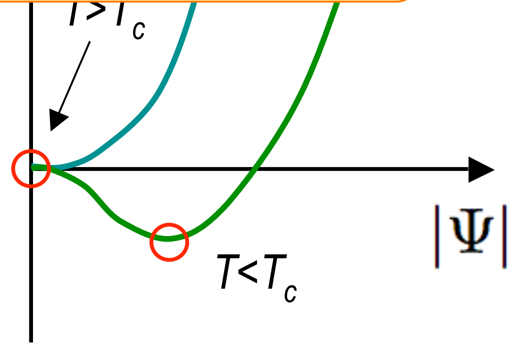
phase ϕ free

spontaneous breaking of *U(1)* symmetry

Free energy expansion at T_c

$$F[\Psi] = a(T - T_c)|\Psi|^2 + b|\Psi|^4$$

$a, b > 0$ scalar under *U(1)*-gauge operation $\Psi \rightarrow \Psi e^{i\chi}$



supplement

Ginzburg-Landau theory

local *U(1)*-gauge invariance

Free energy functional

$$F[\Psi,\vec{A}] = \int d^3r \left[a(T-T_c)|\Psi|^2 + b|\Psi|^4 + K|\vec{D}\Psi|^2 + \frac{(\vec{\nabla}\times\vec{A})^2}{8\pi}\right]$$

$a, b, K > 0$ gradient $\vec{D} = \frac{\hbar}{i}\vec{\nabla} + \frac{2e}{c}\vec{A}$ vector potential $\vec{A}$

variational equations

$$\left\{a + 2b|\Psi|^2 - K\vec{D}^2\right\}\Psi = 0$$

inhomogeneous
oder parameter
structures

domain walls, vortices
etc

$$\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{J}_s$$

supercurrent $\vec{J}_s = \frac{eK}{4\pi\hbar i}\left\{\Psi^*(\vec{D}\Psi) - \Psi(\vec{D}\Psi)^*\right\}$

London equation $\vec{\nabla}^2\vec{B} + \lambda^{-2}\vec{B} = 0$

„massive photon“

Anderson-Higgs mechanism

supplement

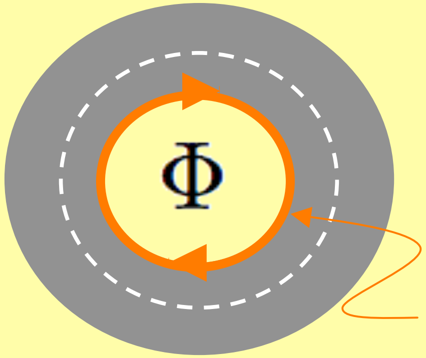
Ginzburg-Landau theory

local *U(1)*-gauge invariance

Free energy functional

single-valued macroscopic wave function

$$\Psi(\vec{r}) = |\Psi(\vec{r})| e^{i\phi(\vec{r})}$$



persistent current
dissipation free

$$\oint d\vec{s} \cdot \vec{J}_S = 0$$

flux quantization

$$\Phi = n\Phi_0 = n\frac{hc}{2e}$$

structures

domain walls, vortices
etc

London equation $\vec{\nabla}^2\vec{B} + \lambda^{-2}\vec{B} = 0$

„massive photon“

Anderson-Higgs mechanism

Microscopic point of view

Cooper pairing of electrons

Microscopic theory of superconductivity

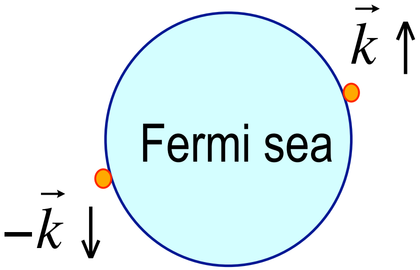
Bardeen-Cooper-Schrieffer (BCS) (1957)

Superconducting state as a coherent state of electron Cooper pairs

$$|\Psi\rangle = \prod_{\vec{k}} |\Phi_{\vec{k}}\rangle \qquad \text{with} \qquad |\Phi_{\vec{k}}\rangle = u_{\vec{k}}|0,0\rangle_{\vec{k}} + v_{\vec{k}}|1,1\rangle_{\vec{k}}$$

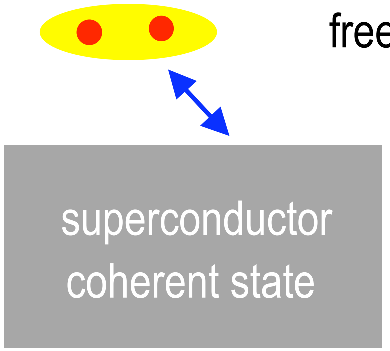
$$\left.\begin{array}{ll} |0,0\rangle_{\vec{k}} & \text{0 electron} \\ |1,1\rangle_{\vec{k}} & \text{1 electron} \end{array}\right\} \text{in each single-electron state} \left\{\begin{array}{l} |+\vec{k}\uparrow\rangle \\ |-\vec{k}\downarrow\rangle \end{array}\right\}$$

Cooper pairs



total momentum

$$\vec{P}_{tot} = 0$$



free adding / removing
of Cooper pairs

number of pairs
not fixed

Microscopic theory of superconductivity

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pair wave function

$$|1,1\rangle_{\vec{k}} = c^{\dagger}_{\vec{k}\uparrow} c^{\dagger}_{-\vec{k}\downarrow} |0,0\rangle_{\vec{k}}$$

$$\Psi_{\vec{k}} = \langle \Psi | c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow} | \Psi \rangle = u_{\vec{k}} v_{\vec{k}}$$

U(1)- gauge operation

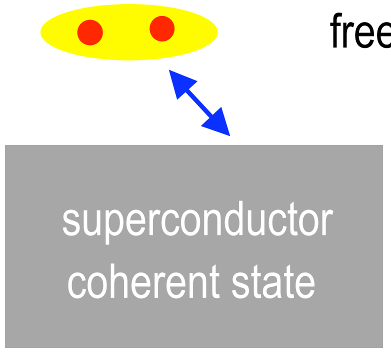
$$c_{\vec{k}s} \rightarrow c_{\vec{k}s} e^{i\alpha}$$

↓

$$\Psi_{\vec{k}} \rightarrow \Psi_{\vec{k}} e^{i2\alpha}$$

order parameter
macroscopic
wavefunction

phase 2α
conjugate to pair number



superconductor
coherent state

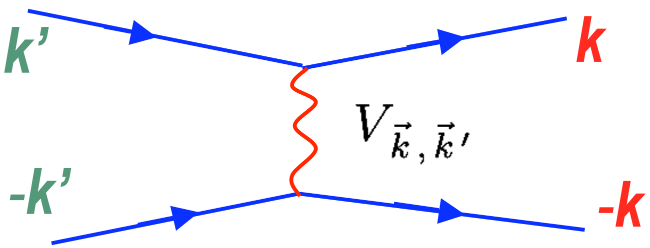
free adding / removing
of Cooper pairs

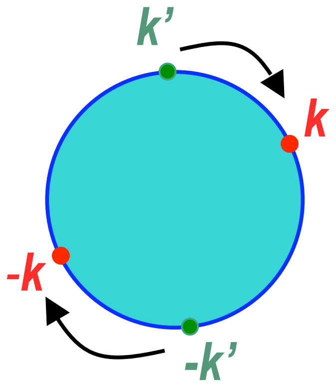
number of pairs
not fixed

Pairing interaction - electron phonon (BCS)

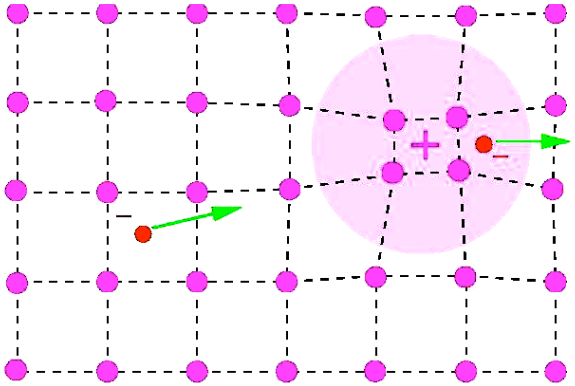
Cooper pair formation (bound state of 2 electrons) needs attractive interaction

$$\mathcal{H}_{pair} = \frac{1}{2\Omega}\sum_{\vec{k},\vec{k}'}\sum_{s,s'} V_{\vec{k},\vec{k}'} c^{\dagger}_{\vec{k},s} c^{\dagger}_{-\vec{k},s'} c_{-\vec{k}',s'} c_{\vec{k}',s}$$





electron phonon interaction:



attractive interaction

↓

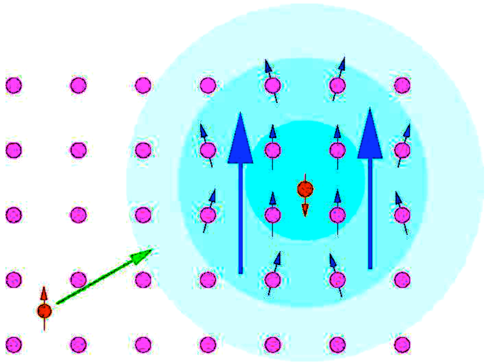
scattering between electron states with degenerate energy

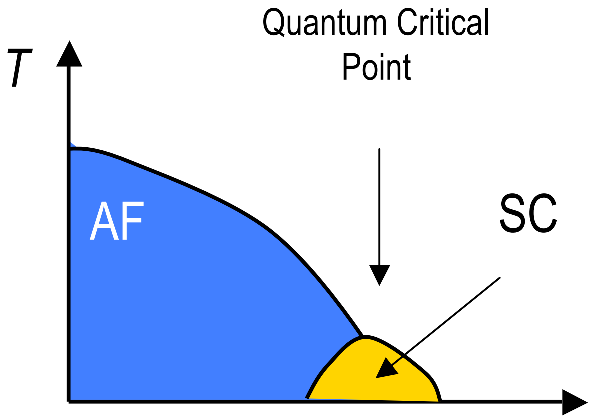
$$\varepsilon_{\vec{k}} = \varepsilon_{-\vec{k}}$$

Alternative mechanism for Cooper pairing

Pairing by magnetic fluctuations:

Berk & Schrieffer (1966)



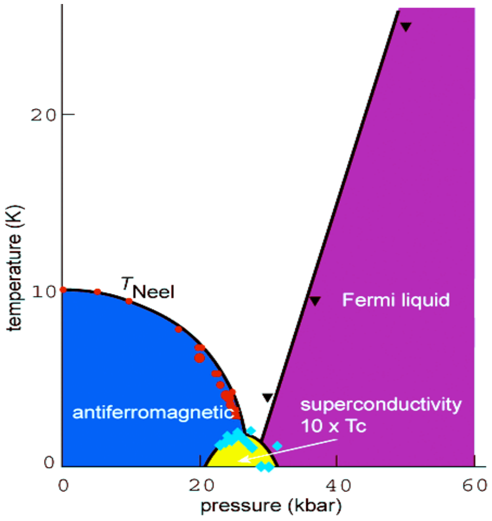


easily spin polarizable medium

longer ranged interaction

⬇

pairing for higher angular momentum

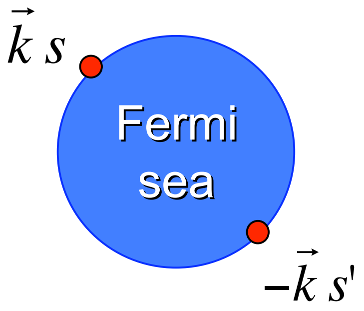


CeIn_3

Cooper pair symmetry

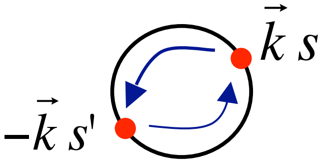
Symmetry of pairs of identical electrons:

$$\Psi_{\vec{k}} = \langle c_{-\vec{k}s'} c_{\vec{k}s} \rangle = \underbrace{\phi(\vec{k})}_{\text{orbital}} \underbrace{\chi_{ss'}}_{\text{spin}}$$



$P_{tot}=0$

Pauli principle:
wave function totally antisymmetric
under particle exchange

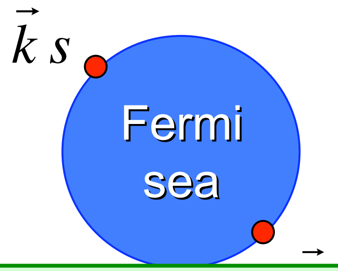


$\vec{k} \rightarrow -\vec{k}$ $\qquad s \leftrightarrow s'$

	angular momentum	spin	
even parity:	$L = 0,2,4,\ldots,$ even	$S=0$ odd	spin singlet
odd parity:	$L = 1,3,5,\ldots,$ odd	$S=1$ even	spin triplet

Cooper pair symmetry

Symmetry of pairs of identical electrons:



$P_{tot}=0$

$$\Psi_{\vec{k}} = \langle c_{-\vec{k}s'} c_{\vec{k}s} \rangle = \phi(\vec{k})\chi_{ss'}$$

Pauli p
wave fu
under p

Classification

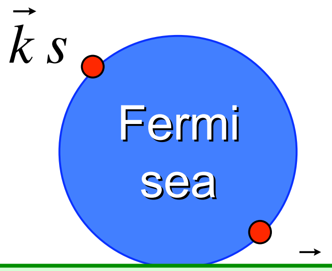
$L = 0,\ S = 0$: most symmetric „conventional pairing“

$L > 0$: lower symmetry „unconventional pairing“

	angular momentum	spin	
even parity:	*L =0,2,4,… ,* even	*S=0* odd	spin singlet
odd parity:	*L = 1,3,5,… ,* odd	*S=1* even	spin triplet

Cooper pair symmetry

Symmetry of pairs of identical electrons:



$P_{tot}=0$

$$\Psi_{\vec{k}} = \langle c_{-\vec{k}s'} c_{\vec{k}s} \rangle = \phi(\vec{k})\chi_{ss'}$$

key symmetries for this classification

time reversal & inversion

Pauli p

wave fu

under p

	angular momentum	spin	
even parity:	$L = 0,2,4,\dots,$ even	$S=0$ odd	spin singlet
odd parity:	$L = 1,3,5,\dots,$ odd	$S=1$ even	spin triplet

Anderson's Theorems (1959,1984)

Cooper pairs with total momentum $P_{tot}=0$ form from degenerate quasiparticle states.

$$|+\vec{k}s\rangle \quad |-\vec{k}s'\rangle$$

with $\epsilon_{\vec{k}s} = \epsilon_{-\vec{k}s'}$

How to guarantee existence of degenerate partners?

- Spin singlet pairing: time reversal symmetry

$$|\vec{k}\uparrow\rangle \rightarrow \hat{T}|\vec{k}\uparrow\rangle = |-\vec{k}\downarrow\rangle \longleftrightarrow \epsilon_{\vec{k}\uparrow} = \epsilon_{-\vec{k}\downarrow}$$

harmful: magnetic impurities, ferromagnetism, Zeemann fields (paramagnetic limiting)

- Spin triplet pairing: inversion symmetry

$$|\vec{k}\uparrow\rangle \rightarrow \hat{I}|\vec{k}\uparrow\rangle = |-\vec{k}\uparrow\rangle \longleftrightarrow \epsilon_{\vec{k}\uparrow} = \epsilon_{-\vec{k}\uparrow}$$

harmful: crystal structure without inversion center

Basic Model
of systems without inversion center

Key symmetries and band structure

Electron state: $|\vec{k}s\rangle$

time reversal: $\hat{T}|\vec{k},s\rangle \rightarrow |-\vec{k},-s\rangle$

inversion: $\hat{I}|\vec{k},s\rangle \rightarrow |-\vec{k},s\rangle$

➡ orbital and spin part distinctly treated

Electron spectrum:

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

charge density spin density

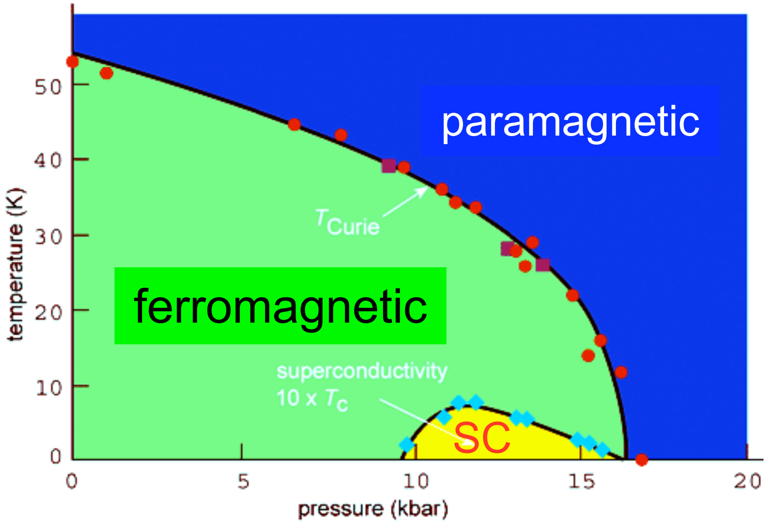
conserved

time reversal $\epsilon_{\vec{k}} = \epsilon_{-\vec{k}}$ $\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$

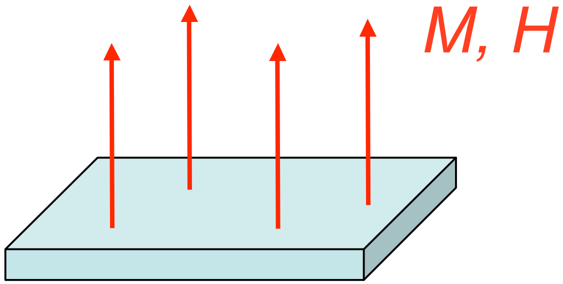
inversion $\epsilon_{\vec{k}} = \epsilon_{-\vec{k}}$ $\vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}$

Lack of time reversal symmetry

UGe$_2$



superconductivity
in a ferromagnet



Electrons in a ferromagnet / magnetic field:

$$\epsilon_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m}$$

and Zeeman field

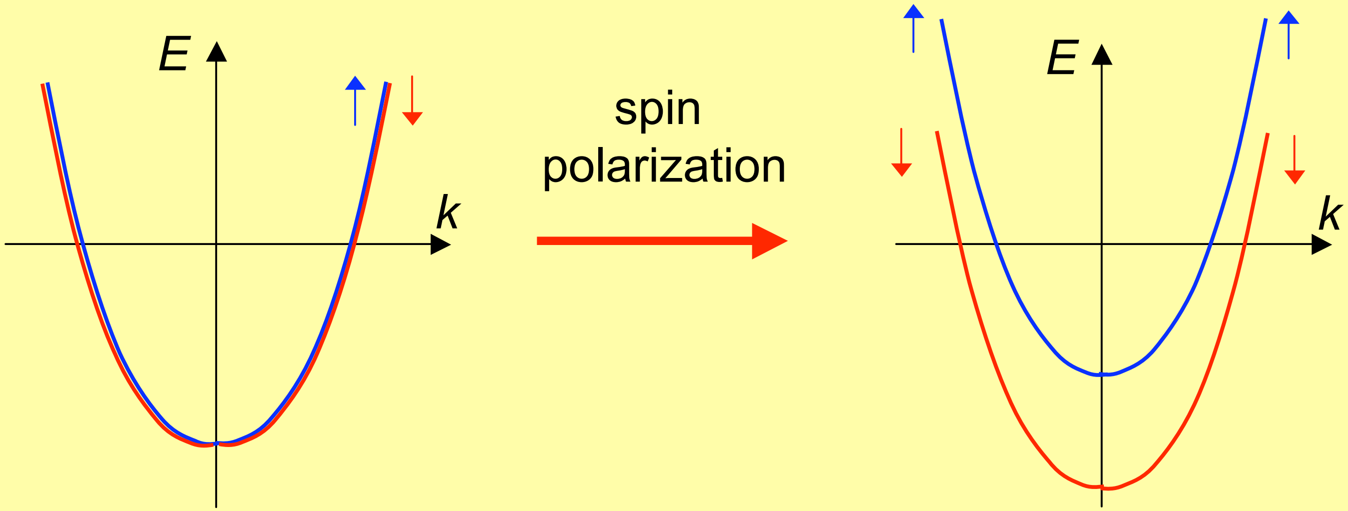
$$-g\mu_B S_z \vec{M}$$
$$-g\mu_B S_z \vec{H}$$

$$\Rightarrow \quad \lambda_{\vec{k}} = \begin{cases} -g\mu_B \frac{\hbar}{2} M \\ -g\mu_B \frac{\hbar}{2} H \end{cases} \quad \Rightarrow \quad \vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}$$

Band structure - band splitting

$$\mathcal{H} = \sum_{\vec{k},s} \left(\frac{\hbar^2 \vec{k}^2}{2m} - \mu \right) c^{\dagger}_{\vec{k}+s} c_{\vec{k}+s} + \alpha \sum_{\vec{k},s,s'} \hat{z} \cdot (c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'})$$

Spin split energy spectrum: $$E_{\vec{k}\pm} = \frac{\hbar^2 \vec{k}^2}{2m} - \mu \pm \alpha$$

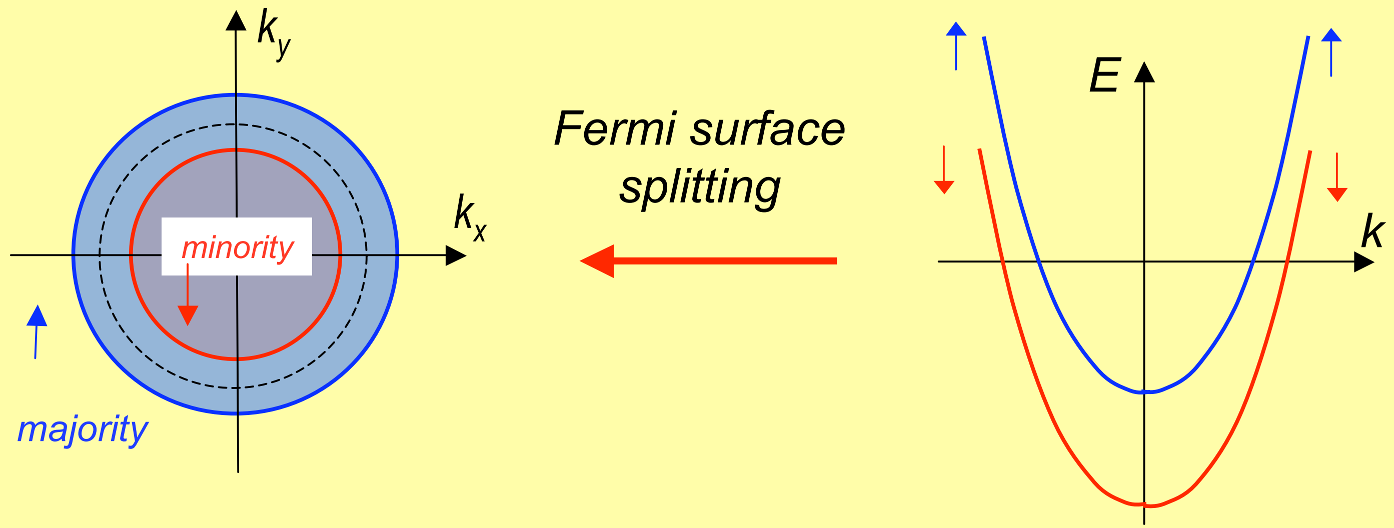


Band structure - band splitting

$$\mathcal{H} = \sum_{\vec{k},s}\left(\frac{\hbar^2\vec{k}^2}{2m}-\mu\right) c^\dagger_{\vec{k}+s}c_{\vec{k}+s} + \alpha\sum_{\vec{k},s,s'}\hat{z}\cdot\left(c^\dagger_{\vec{k}s}\vec{\sigma}_{ss'}c_{\vec{k}s'}\right)$$

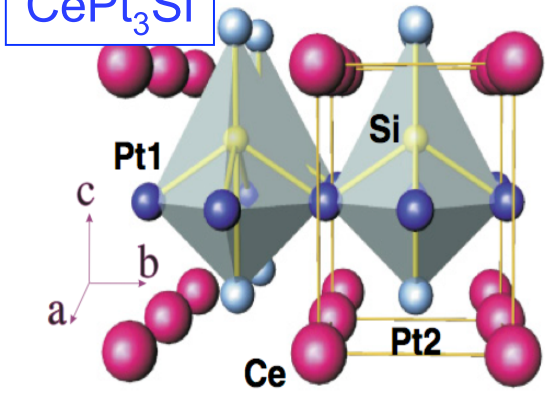
Spin split energy spectrum:

$$E_{\vec{k}\pm} = \frac{\hbar^2\vec{k}^2}{2m} - \mu \pm \alpha$$

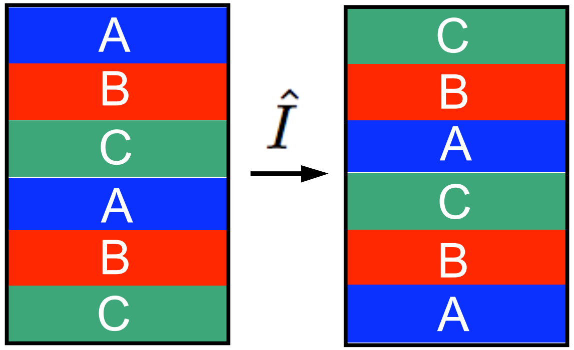


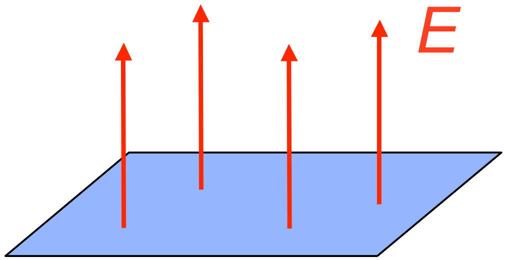
Lack of inversion symmetry - non-centrosymmetric

$CePt_3Si$



C_{4v}





motion of electron in electric field:

$$\epsilon_{\vec{k}} = \frac{\hbar^2 \vec{k}^2}{2m}$$

and special relativity

$$\vec{B} = -\frac{\vec{v}_{\vec{k}}}{c} \times \vec{E} = \frac{\hbar E}{mc}(\vec{k} \times \hat{z})$$

spin-orbit coupling:

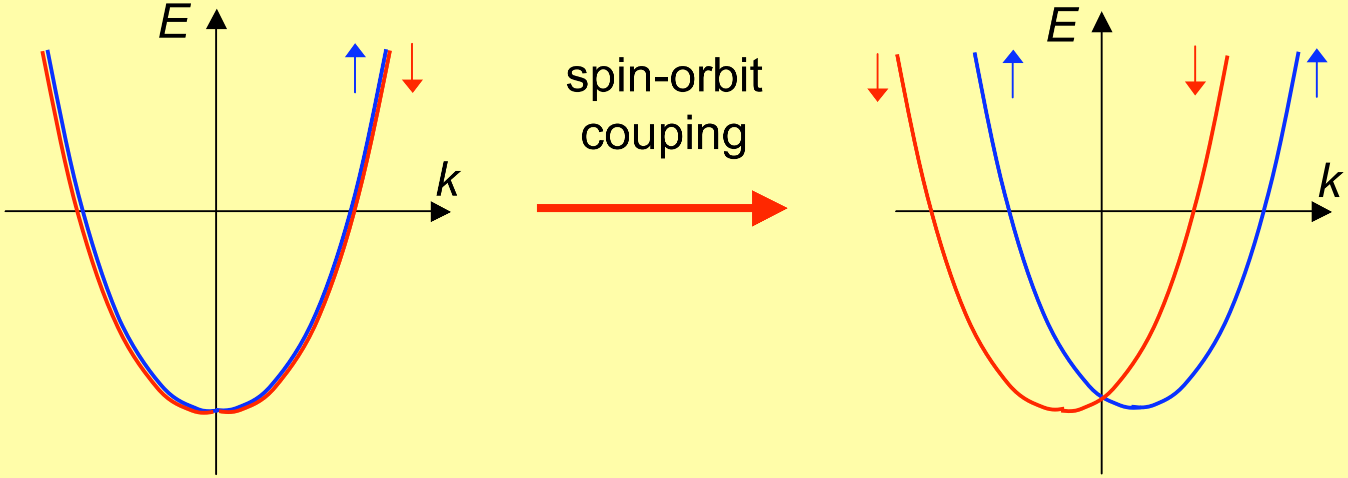
$$-\mu_B \vec{B} \cdot \vec{S} = \frac{\hbar \mu_B E}{mc}(\vec{k} \times \hat{z}) \cdot \vec{S} = \alpha' \vec{\lambda}_{\vec{k}} \cdot \vec{S}$$

Rashba-like spin-orbit coupling $\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$

Band structure - band splitting

$$\mathcal{H} = \sum_{\vec{k},s} \left(\frac{\hbar^2 k^2}{2m} - \mu \right) c^\dagger_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} (\vec{k} \times \hat{z})(c^\dagger_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'})$$

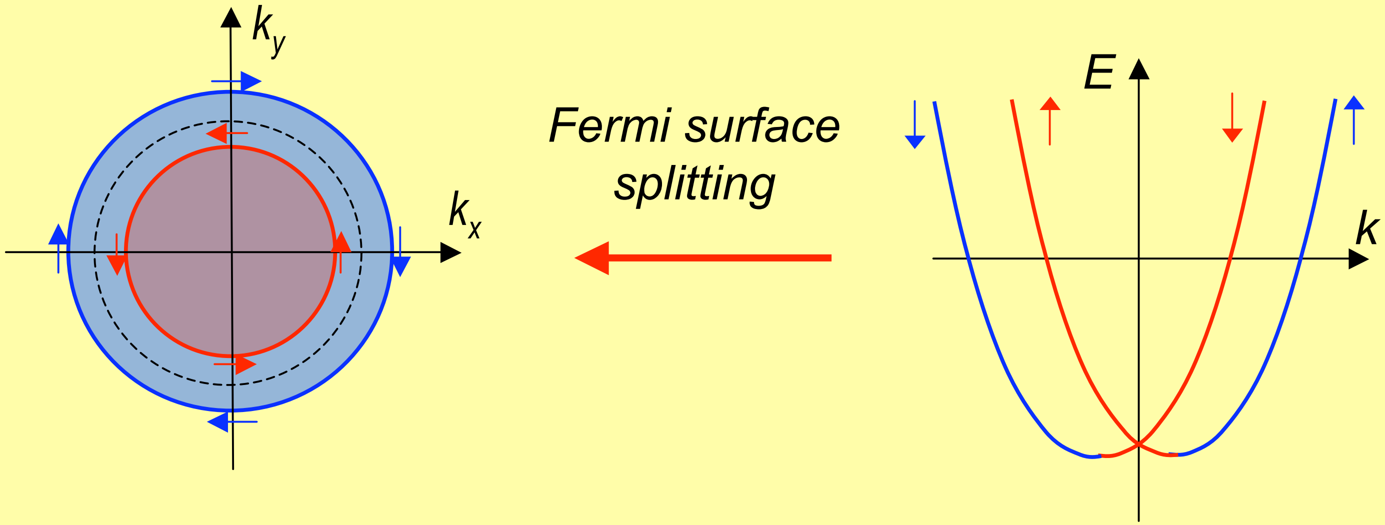
Spin split energy spectrum: $$E_{\vec{k}\pm} = \frac{\hbar^2 k^2}{2m} - \mu \pm \alpha |\vec{k} \times \hat{z}|$$



Band structure - band splitting

$$\mathcal{H} = \sum_{\vec{k},s} \left(\frac{\hbar^2 k^2}{2m} - \mu \right) c^\dagger_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} (\vec{k} \times \hat{z})(c^\dagger_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'})$$

Spin split energy spectrum: $$E_{\vec{k}\pm} = \frac{\hbar^2 k^2}{2m} - \mu \pm \alpha |\vec{k} \times \hat{z}|$$



Fermi surface splitting

Superconducting phase

Superconducting phases

pair wave function $\hat{\Psi}_{\vec{k}} = \begin{pmatrix} \Psi_{\vec{k}\uparrow\uparrow} & \Psi_{\vec{k}\uparrow\downarrow} \\ \Psi_{\vec{k}\downarrow\uparrow} & \Psi_{\vec{k}\downarrow\downarrow} \end{pmatrix}$

spin singlet, even parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix}$$

$$= i\hat{\sigma}^y \psi(\vec{k})$$

1 configuration $\frac{\psi(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\psi(-\vec{k}) = \psi(\vec{k})$$

spin triplet, odd parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

$$= i\vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\hat{\sigma}^y$$

3 configurations $\begin{cases} (-d_x(\vec{k}) + id_y(\vec{k}))|\uparrow\uparrow\rangle \\ \frac{d_z(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ (d_x(\vec{k}) + id_y(\vec{k}))|\downarrow\downarrow\rangle \end{cases}$

$$\vec{d}(-\vec{k}) = -\vec{d}(\vec{k})$$

Superconducting phases

pair wave function $\hat{\Psi}_{\vec{k}} = \begin{pmatrix} \Psi_{\vec{k}\uparrow\uparrow} & \Psi_{\vec{k}\uparrow\downarrow} \\ \Psi_{\vec{k}\downarrow\uparrow} & \Psi_{\vec{k}\downarrow\downarrow} \end{pmatrix}$

spin singlet, even parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} 0 & \psi(\vec{k}) \\ -\psi(\vec{k}) & 0 \end{pmatrix}$$

$$= i\hat{\sigma}^y \psi(\vec{k})$$

1 configuration $\frac{\psi(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

$$\psi(-\vec{k}) = \psi(\vec{k})$$

spin triplet, odd parity

$$\hat{\Psi}_{\vec{k}} = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

$$= i\vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\hat{\sigma}^y$$

3 configurations $\begin{cases} (-d_x(\vec{k}) + id_y(\vec{k}))|\uparrow\uparrow\rangle \\ \frac{d_z(\vec{k})}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\ (d_x(\vec{k}) + id_y(\vec{k}))|\downarrow\downarrow\rangle \end{cases}$

$$\vec{d} \perp \vec{S}$$

$$\vec{d}(-\vec{k}) = -\vec{d}(\vec{k})$$

supplement

Anderson theorem for small perturbation

superconducting phase: bare T_c = T_{c0} and $|\alpha \vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

singlet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left\langle |\psi(\vec{k})|^2 f(\rho_{\vec{k}}) \left\{ 1 + \hat{\lambda}_{\vec{k}} \cdot \hat{\lambda}_{-\vec{k}} \right\} \right\rangle_{\vec{k}}$$

$$T_c = T_{c0}$$
$$\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$$

no inversion

$$T_c \neq T_{c0}$$
$$\vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}$$

no time reversal

$$f(\rho) = Re \sum_{n=1}^{\infty} \left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1} \right) \qquad \rho_{\vec{k}} = \frac{|\vec{\lambda}_{\vec{k}}|}{\pi k_B T_c} \qquad \hat{\lambda}_{\vec{k}} = \frac{\vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

supplement

Anderson theorem for small perturbation

superconducting phase: bare T_c = T_{c0} and $|\alpha\vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

triplet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left\langle 2f(\rho_{\vec{k}})\left\{(\hat{\lambda}_{\vec{k}}\cdot\vec{d}^*(\vec{k}))(\hat{\lambda}_{-\vec{k}}\cdot\vec{d}(\vec{k})) + |\vec{d}(\vec{k})|^2\left[1-\hat{\lambda}_{\vec{k}}\cdot\hat{\lambda}_{-\vec{k}}\right]\right\}\right\rangle_{\vec{k}}$$

$$\overbrace{\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}} \qquad\qquad \vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}}^{T_c = T_{c0}}$$

no inversion — $\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

no time reversal — $\vec{\lambda}_{\vec{k}} \perp \vec{d}(\vec{k})$

spin structure adapted to perturbation

$$T_c \neq T_{c0}$$

otherwise

$$f(\rho) = Re\sum_{n=1}^{\infty}\left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1}\right) \qquad \rho_{\vec{k}} = \frac{|\vec{\lambda}_{\vec{k}}|}{\pi k_B T_c} \qquad \hat{\lambda}_{\vec{k}} = \frac{\vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

supplement

Anderson theorem for small perturbation

superconducting phase: bare T_c = T_{c0} and $|\alpha \vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

triplet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \Big\langle 2f(\rho_{\vec{k}})\Big\{(\hat{\lambda}_{\vec{k}}\cdot\vec{d}^*(\vec{k}))(\hat{\lambda}_{-\vec{k}}\cdot\vec{d}(\vec{k})) +$$

$$\underbrace{T_c = T_{c0}}$$

$$\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$$

no inversion

$$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$$

$$\vec{\lambda}_{\vec{k}} = +\vec{\lambda}_{-\vec{k}}$$

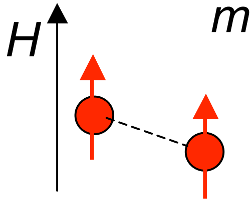
no time reversal

$$\vec{\lambda}_{\vec{k}} \perp \vec{d}(\vec{k})$$

spin structure adapted to perturbation

otherwise

equal spin pairing parallel to magnetic field



$$\vec{d}(\vec{k}) \perp \vec{H}$$

$$f(\rho) = Re\sum_{n=1}^{\infty}\left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1}\right) \qquad \rho_{\vec{k}} = \frac{|\vec{\lambda}_{\vec{k}}|}{\pi k_B T_c} \qquad \hat{\lambda}_{\vec{k}} = \frac{\vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

supplement

Anderson theorem for small perturbation

superconducting phase: bare T_c = T_{c0} and $|\alpha\vec{\lambda}_{\vec{k}}| \sim k_B T_{c0}$

triplet pairing

$$\ln\left(\frac{T_c}{T_{c0}}\right) = \left\langle 2f(\rho_{\vec{k}}) \left\{ (\hat{\lambda}_{\vec{k}} \cdot \vec{d}^*(\vec{k}))(\hat{\lambda}_{-\vec{k}} \cdot \vec{d}(\vec{k})) + |\vec{d}(\vec{k})|^2 \left[1 - \hat{\lambda}_{\vec{k}} \cdot \hat{\lambda}_{-\vec{k}} \right] \right\} \right\rangle_{\vec{k}}$$

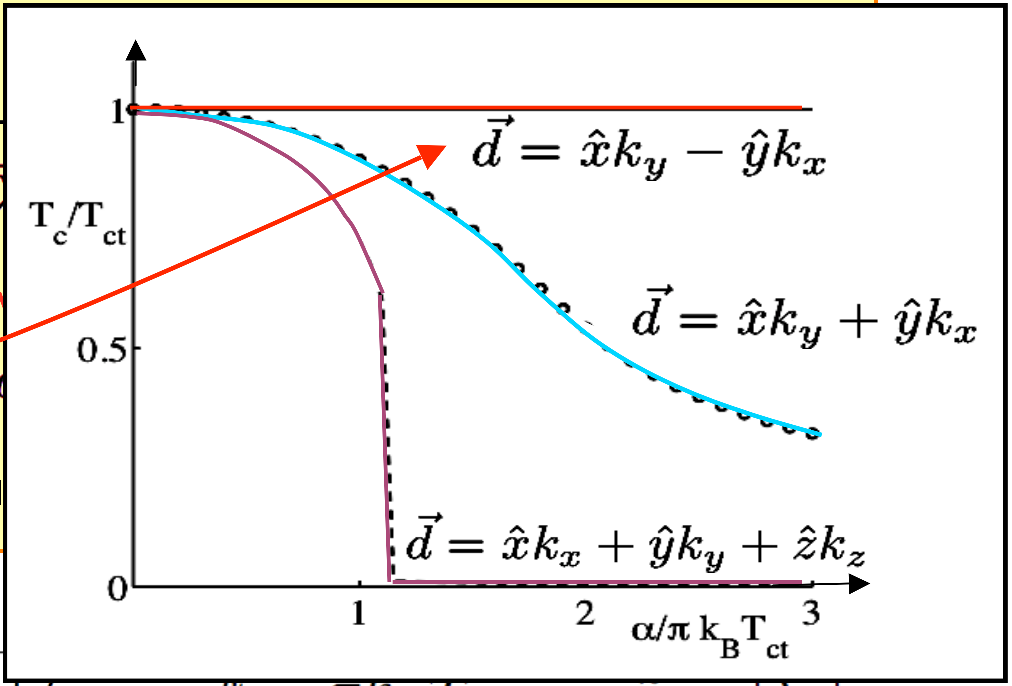
$$T_c = T_{c0}$$

$\vec{\lambda}_{\vec{k}} = -\vec{\lambda}_{-\vec{k}}$ $\vec{\lambda}_{\vec{k}} = +\vec{\lambda}$

no inversion no time re

$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$ $\vec{\lambda}_{\vec{k}} \perp$

spin structure adapted to pertu



$$f(\rho) = Re \sum_{n=1}^{\infty} \left(\frac{1}{2n-1+i\rho} - \frac{1}{2n-1} \right)$$

$\pi k_B T_c$ $|\vec{\lambda}_{\vec{k}}|$

Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

time reversal symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = -\mu_B \vec{H}$$

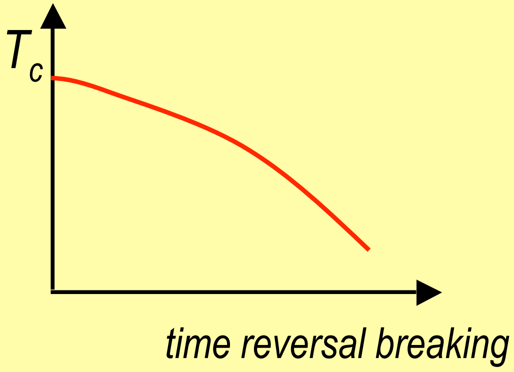
Zeeman coupling

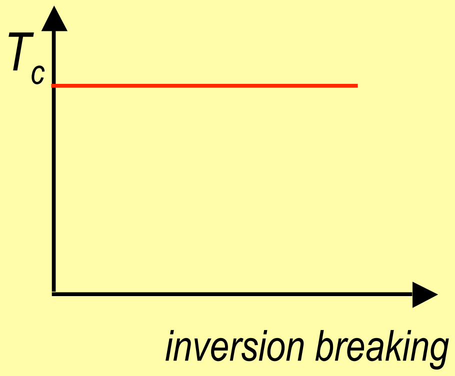
inversion symmetry breaking

$$\alpha \vec{\lambda}_{\vec{k}} = \alpha (\vec{k} \times \hat{z})$$

Rashba spin-orbit coupling

spin-singlet even-parity pairing





Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

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$$\alpha \vec{\lambda}_{\vec{k}} = -\mu_B \vec{H}$$

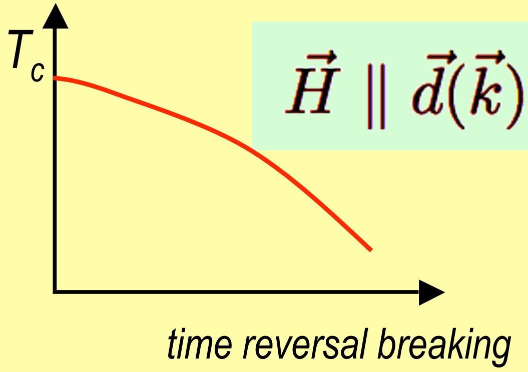
Zeeman coupling

inversion symmetry breaking

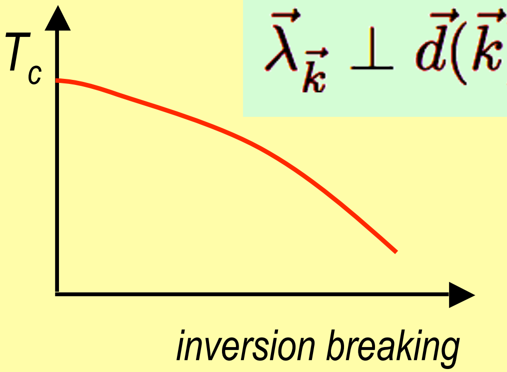
$$\alpha \vec{\lambda}_{\vec{k}} = \alpha (\vec{k} \times \hat{z})$$

Rashba spin-orbit coupling

spin-triplet odd-parity pairing



$$\vec{H} \parallel \vec{d}(\vec{k})$$



$$\vec{\lambda}_{\vec{k}} \perp \vec{d}(\vec{k})$$

Anderson theorem for perturbations

$$\mathcal{H} = \sum_{\vec{k},s} (\epsilon_{\vec{k}} - \mu) c^{\dagger}_{\vec{k}s} c_{\vec{k}s} + \alpha \sum_{\vec{k},s,s'} \vec{\lambda}_{\vec{k}} \cdot \{ c^{\dagger}_{\vec{k}s} \vec{\sigma}_{ss'} c_{\vec{k}s'} \}$$

time reversal symmetry breaking

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Zeeman coupling

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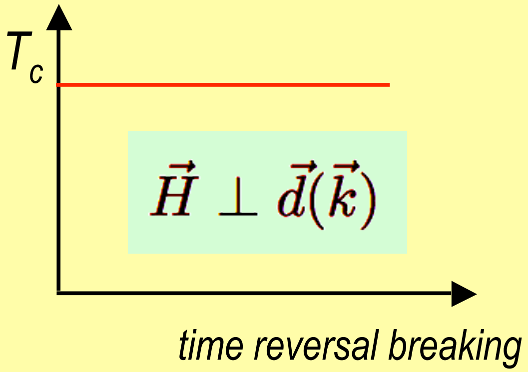
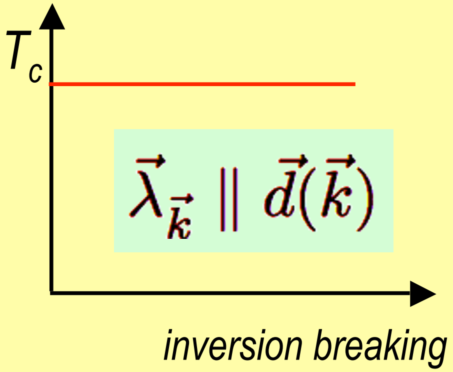
Rashba spin-orbit coupling

spin-triplet odd-parity pairing



$$\vec{H} \perp \vec{d}(\vec{k})$$

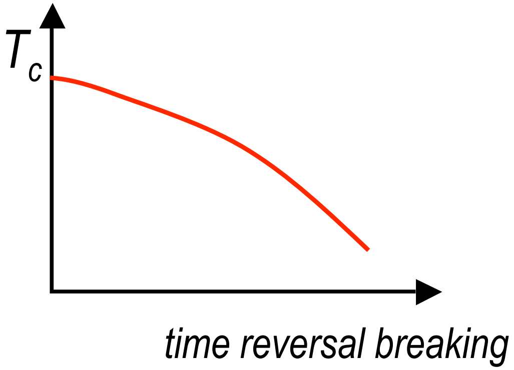


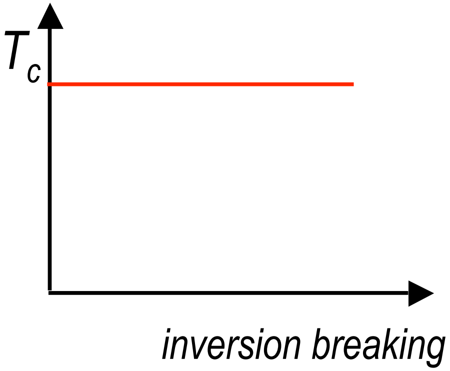
$$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$$

Structure of pairing state

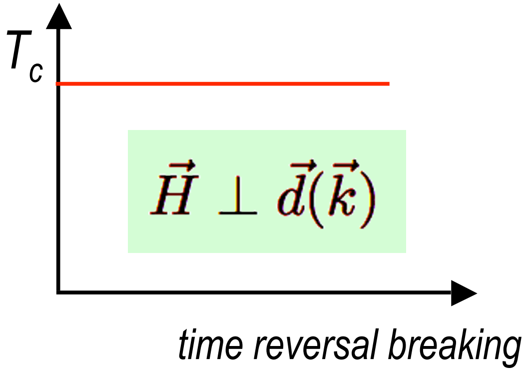
Anderson theorem for perturbations - summary

spin singlet even parity

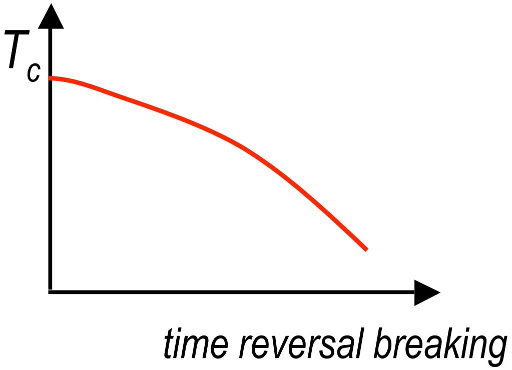


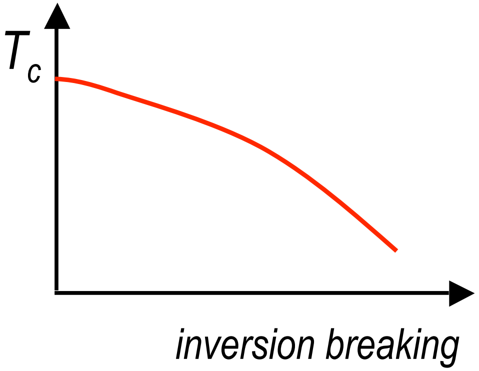


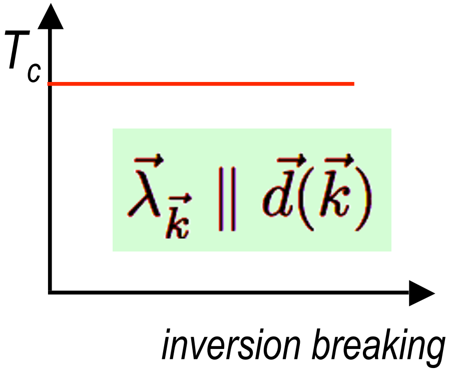
spin triplet odd parity



$\vec{H} \perp \vec{d}(\vec{k})$



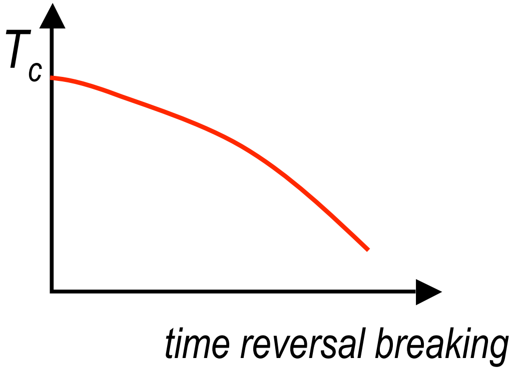


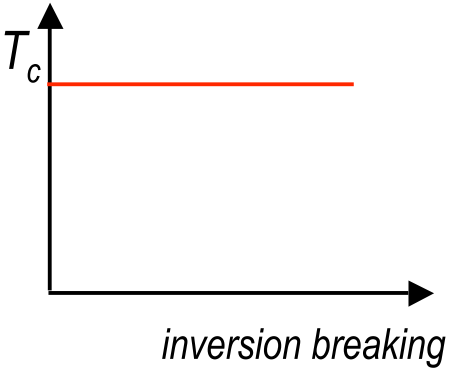


$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

Anderson theorem for perturbations - summary

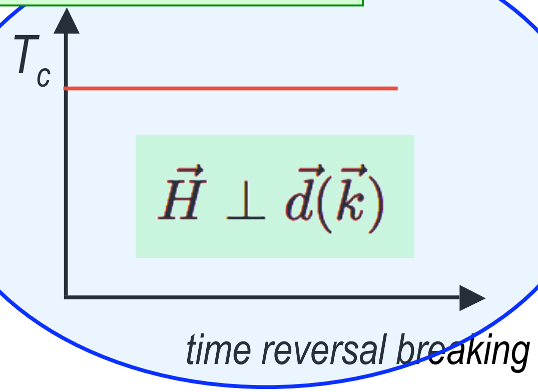
spin singlet even parity



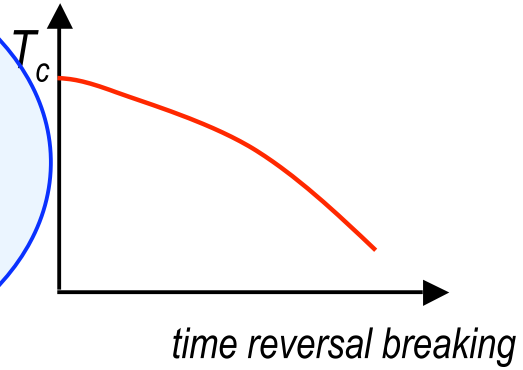


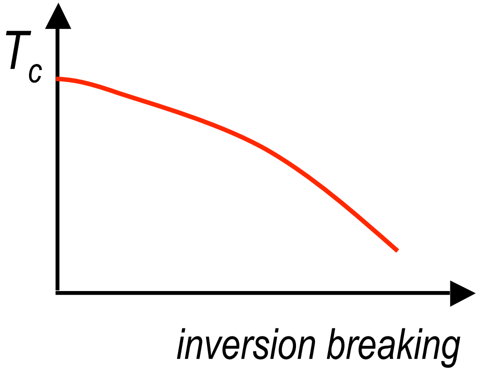
spin triplet odd parity

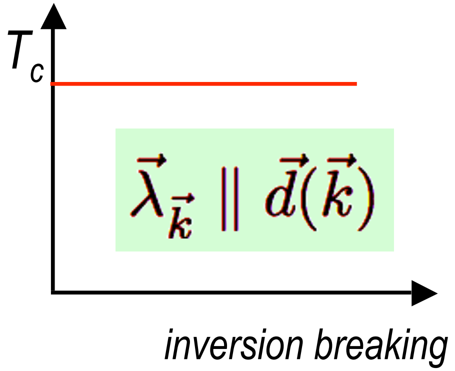
broken time reversal symmetry



$\vec{H} \perp \vec{d}(\vec{k})$

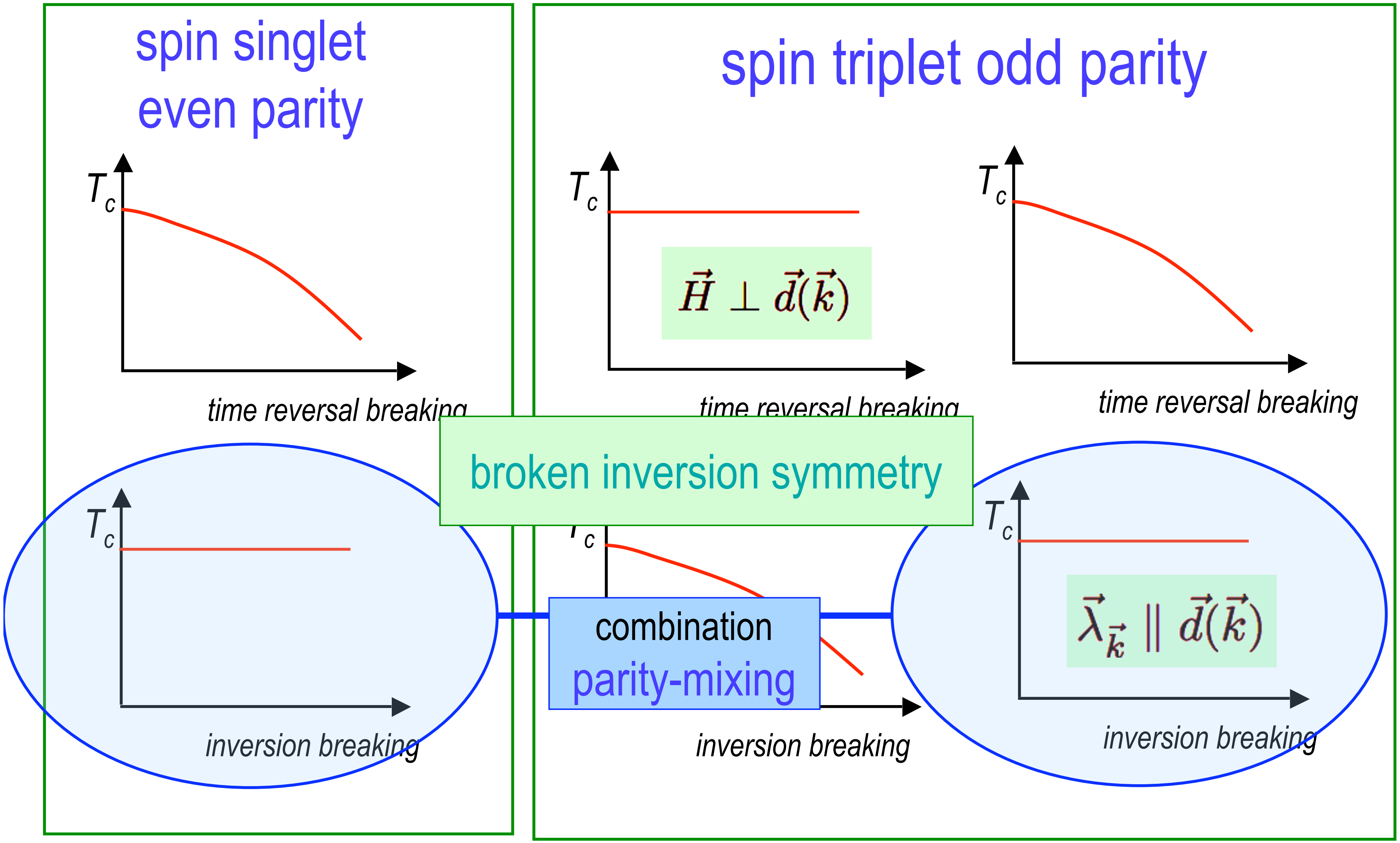






$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

Anderson theorem for perturbations - summary



spin singlet
even parity

spin triplet odd parity

$\vec{H} \perp \vec{d}(\vec{k})$

broken inversion symmetry

combination
parity-mixing

$\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

Structure of pairing states

time reversal symmetry broken (e.g. magnetic field)

spin triplet state: $\hat{\Psi}_{\vec{k}} = i\vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\hat{\sigma}^y$

with spin
parallel to field

Cooper pair
spin expectation value $\vec{d}^* \times \vec{d} \propto \langle \vec{S} \rangle \neq 0$

$\vec{d}(\vec{k}) \perp \vec{H}$

inversion symmetry broken (e.g. non-centrosymmetric crystal)

combination of spin singlet and spin triplet state: *mixed parity state*

$$\hat{\Psi}_{\vec{k}} = \left\{ \psi(\vec{k}) + \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right\} i\hat{\sigma}^y$$

with $\vec{\lambda}_{\vec{k}} \parallel \vec{d}(\vec{k})$

Mixed parity states are *non-unitary*

unitary superconducting states: $\hat{\Psi}_{\vec{k}}\hat{\Psi}^{\dagger}_{\vec{k}} = |\Psi_{\vec{k}}|^2\hat{\sigma}_0 \propto$ *2x2 unit matrix*

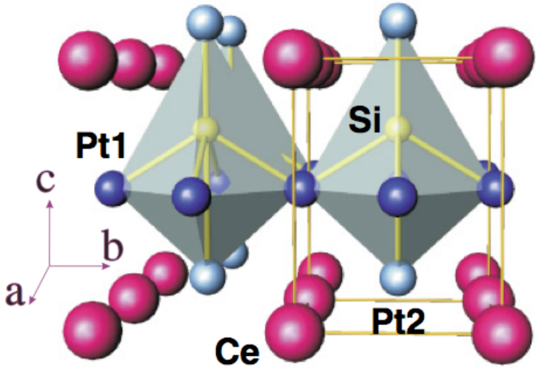
$$\hat{\Psi}_{\vec{k}} = \left\{\psi(\vec{k}) + \vec{d}(\vec{k})\cdot\hat{\vec{\sigma}}\right\} i\hat{\sigma}^y$$

$$\hat{\Psi}_{\vec{k}}\hat{\Psi}^{\dagger}_{\vec{k}} = (|\psi|^2 + |\vec{d}|^2)\hat{\sigma}_0 + \underbrace{\{\psi^*\vec{d} + \psi\vec{d}^*\}}_{\propto\, \vec{\lambda}_{\vec{k}}}\cdot\hat{\vec{\sigma}} + i\underbrace{\{\vec{d}\times\vec{d}^*\}}_{\propto\, \vec{H}}\cdot\hat{\vec{\sigma}}$$

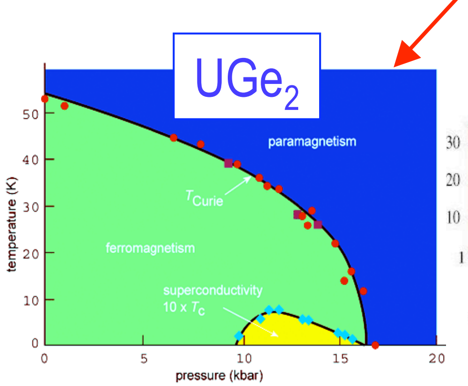
inversion symmetry violated

time reversal symmetry violated

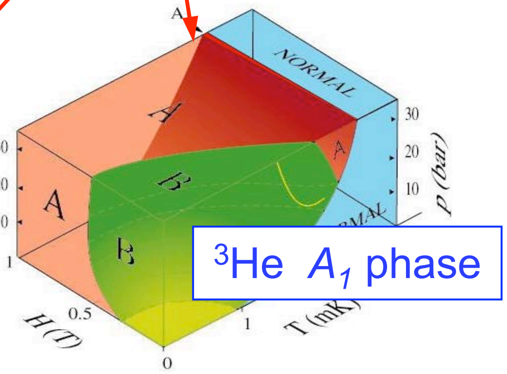
CePt$_3$Si



UGe$_2$



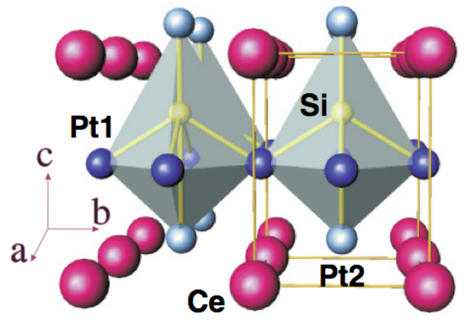
^{3}He A_1 phase



Non-centrosymmetric superconductors

Ce-based heavy Fermion superconductors

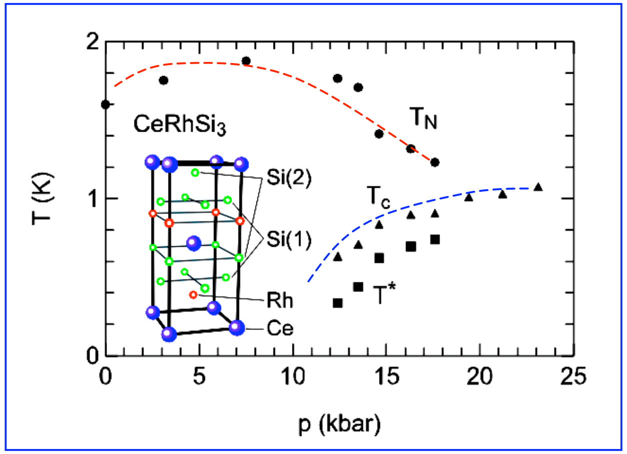
$CePt_3Si$



ambient pressure

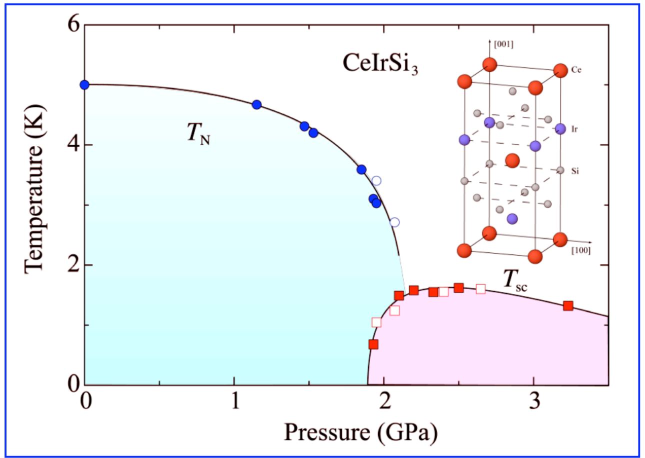
Bauer et al (2004)

$CeRhSi_3$



Kimura et al. (2005)

$CeIrSi_3$

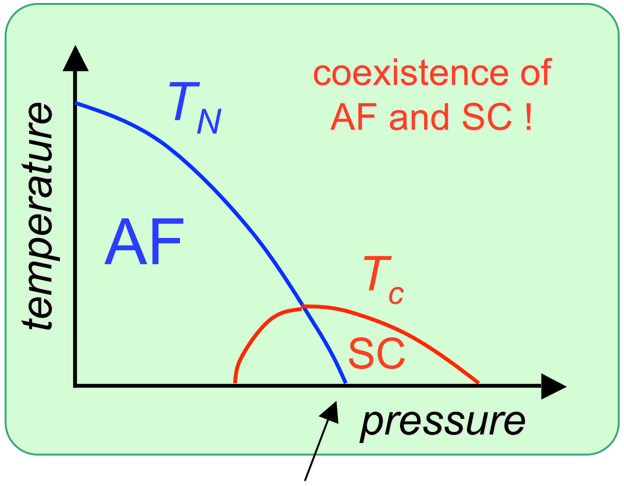


Onuki et al. (2005)

Rashba-type of spin-orbit coupling

$$\vec{\lambda}_{\vec{k}} = \vec{k} \times \hat{z} = \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}$$

tetragonal crystal lattice



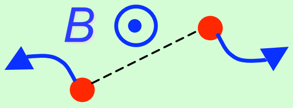
quantum critical point

Upper critical field

How to destroy Cooper pairs by a magnetic field?

Two types of depairing

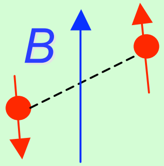
- *orbital depairing*



$$H_{c2}^{orb} \sim \frac{\Phi_0}{2\pi\xi_0^2}$$

coherence length: $\xi_0 \sim \frac{\hbar v_F}{\pi k_B T_c}$

- *paramagnetic depairing*



$$\underset{\text{magnetic energy}}{\frac{\chi_p}{2} H_p^2} \sim \underset{\text{condensation energy}}{\frac{H_c^2}{8\pi}}$$

$$H_p \sim \frac{H_c}{\sqrt{4\pi\chi_p}} \sim \frac{k_B T_c}{\mu_B}$$

Heavy Fermion superconductors:

effective electron mass: $m^* \gg m_e$

„very slow Fermions“

$$\xi_0 \sim 20 - 100\text{Å}$$

➡ orbital depairing mechanism weak

paramagnetic limiting important !?

strong magnetic correlations/order

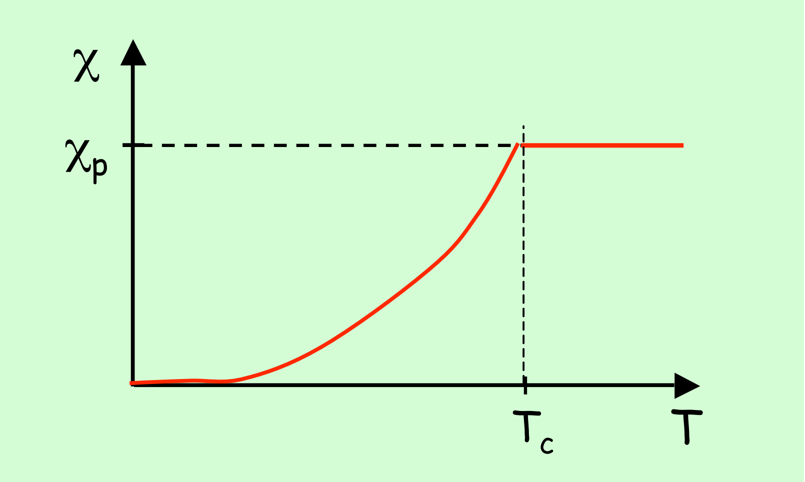
⬇

unconventional pairing likely

Spin polarization - spin susceptibility

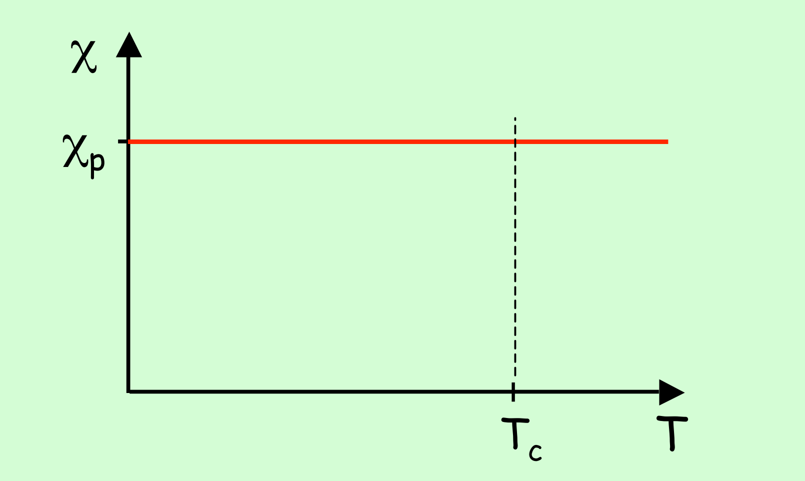
- spin singlet pairing ⟶ Yosida behavior of spin susceptibility

pair breaking
by spin polarization



- spin triplet pairing

no pair breaking
for equal-spin pairing



χ = const. for $\vec{d}(\vec{k})\cdot\vec{H}=0$

note: $\vec{S}\parallel\vec{H}$ with $\vec{d}\perp\vec{S}$

Paramagnetic limiting field

destruction of superconductivity due to Zeeman splitting of electron spins

Compare the two energies at *T=0K*

superconducting condensation energy

$$E_{cond} = \frac{H_c(0)^2}{8\pi}$$

thermodynamic critical field

paramagnetic energy

$$E_{para} = \frac{1}{2}\{\chi_P - \chi(0)\}H^2$$

Pauli susceptibility

spin susceptibility at *T=0K*

paramagnetic limiting field

$$H_p = \frac{H_c(0)}{\sqrt{4\pi(\chi_p - \chi(0))}}$$

Paramagnetic limiting field

destruction of superconductivity due to Zeeman splitting of electron spin states

Compare the two energies at *T=0K*

thermodynamic critical field

superconducting condensation energy

$$E_{cond} = \frac{H_c(0)^2}{8\pi}$$

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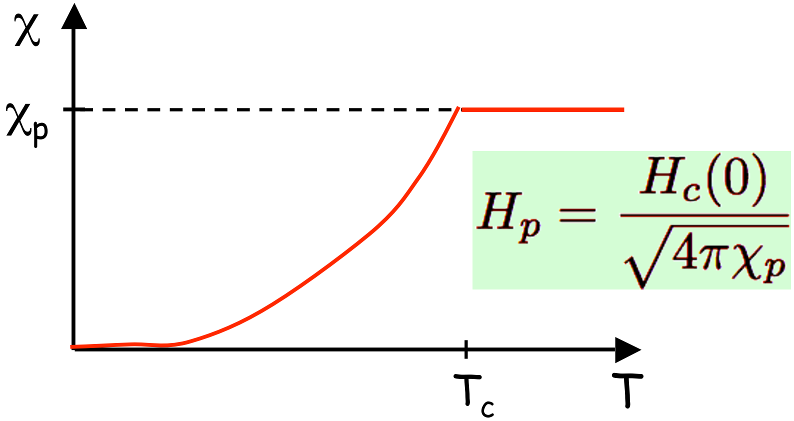
spin singlet pairing



$$H_p = \frac{H_c(0)}{\sqrt{4\pi\chi_p}}$$

Paramagnetic limiting field

destruction of superconductivity due to Zeeman splitting of electron spin states

Compare the two energies at *T=0K*

thermodynamic critical field

superconducting condensation energy

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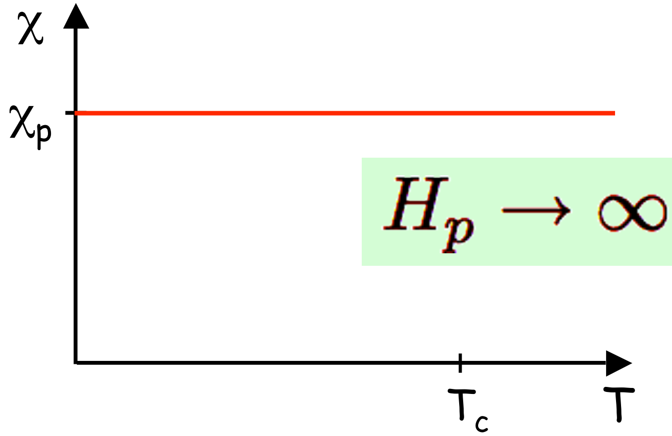
Pauli susceptibility

spin susceptibility at *T=0K*

paramagnetic limiting field

$$H_p = \frac{H_c(0)}{\sqrt{4\pi(\chi_p - \chi(0))}}$$

spin triplet pairing



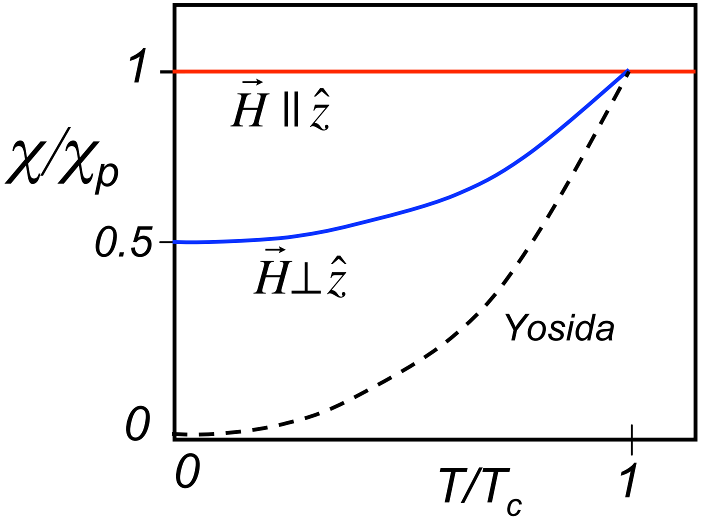
$$H_p \rightarrow \infty$$

Paramagnetic limiting field - mixed parity state

spin susceptibility of non-centrosymmetric SC

$$\vec{\lambda}_{\vec{k}} = \vec{k} \times \hat{z} = \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} \Rightarrow$$

as in $CePt_3Si$, $CeRhSi_3$, $CeIrSi_3$



paramagnetic limiting field

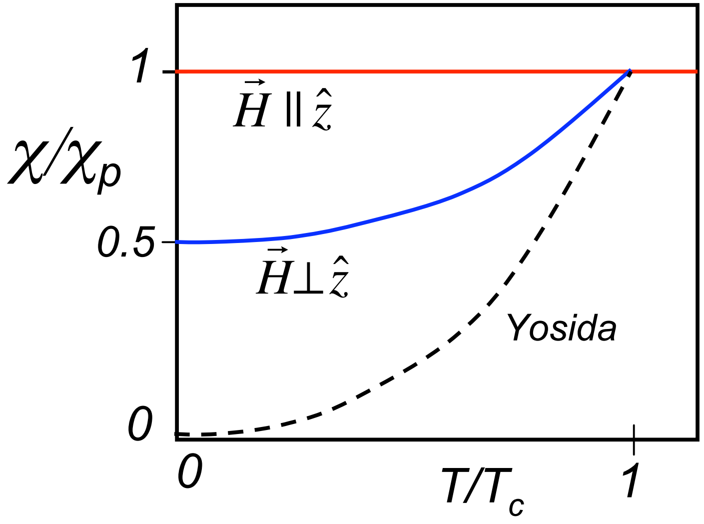
$$H_p = \frac{H_c(0)}{\sqrt{4\pi(\chi_p - \chi(0))}}$$

$\vec{H} \perp \hat{z}$	paramagnetic limiting
$\vec{H} \parallel \hat{z}$	no paramagn. limiting

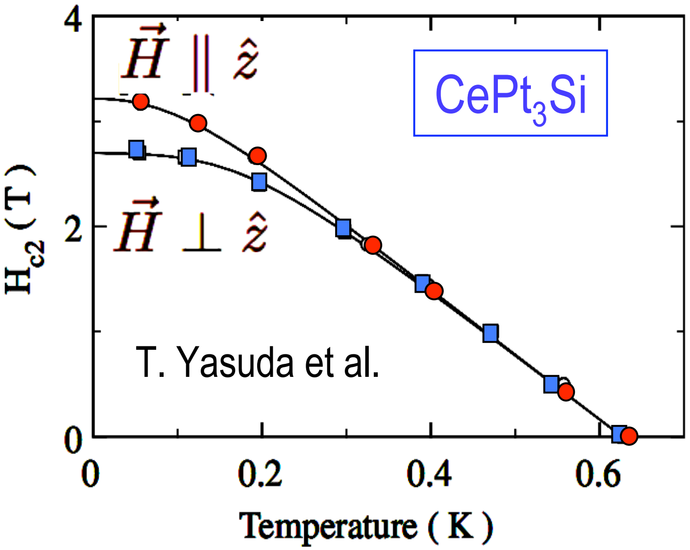
Paramagnetic limiting field - mixed parity state

spin susceptibility of non-centrosymmetric SC

$$\vec{\lambda}_{\vec{k}} = \vec{k} \times \hat{z} = \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} \Rightarrow$$



as in *CePt$_3$Si, CeRhSi$_3$, CeIrSi$_3$*



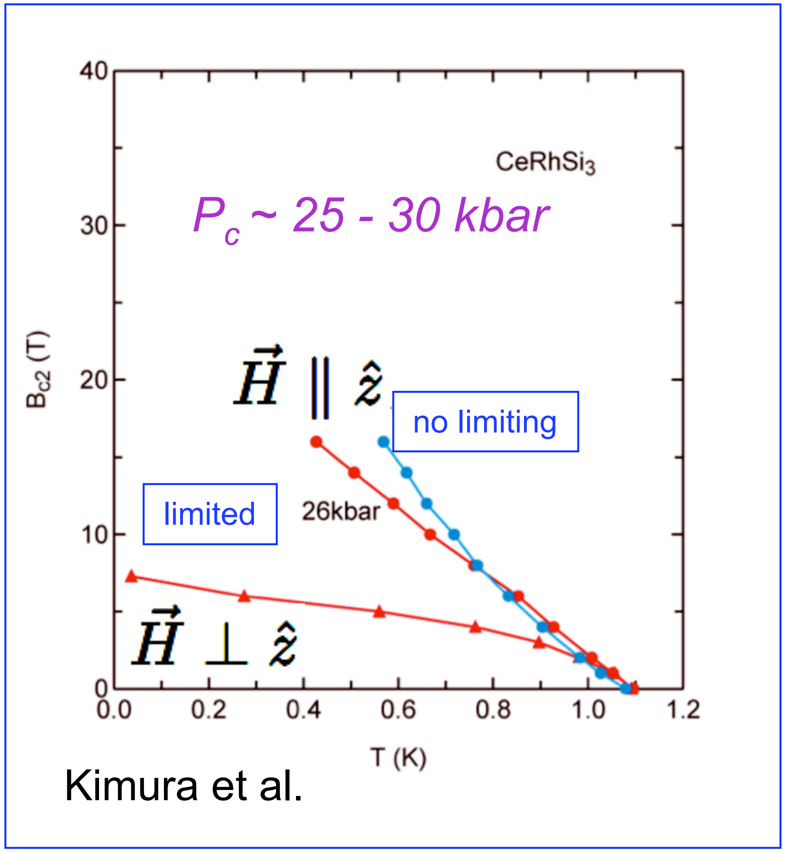
does not follow the expectations !

$\vec{H} \perp \hat{z}$ paramagnetic limiting

$\vec{H} \parallel \hat{z}$ no paramagn. limiting

Upper critical field and paramagnetic limiting

$CeRhSi_3$



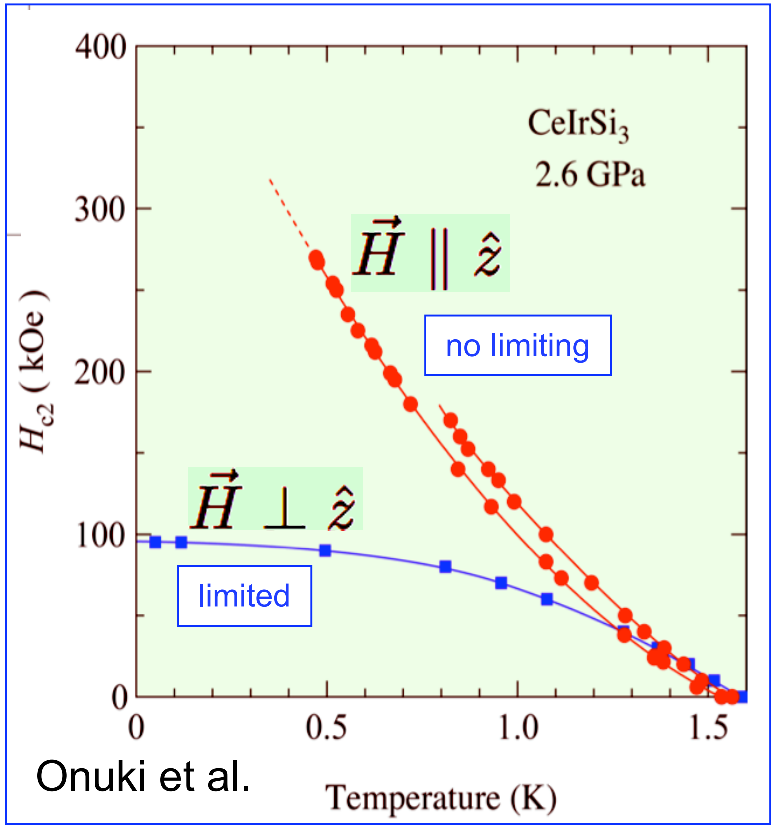
$P_c \sim 25 - 30$ kbar

$\vec{H} \parallel \hat{z}$ no limiting

$\vec{H} \perp \hat{z}$ limited

Kimura et al.

$CeIrSi_3$



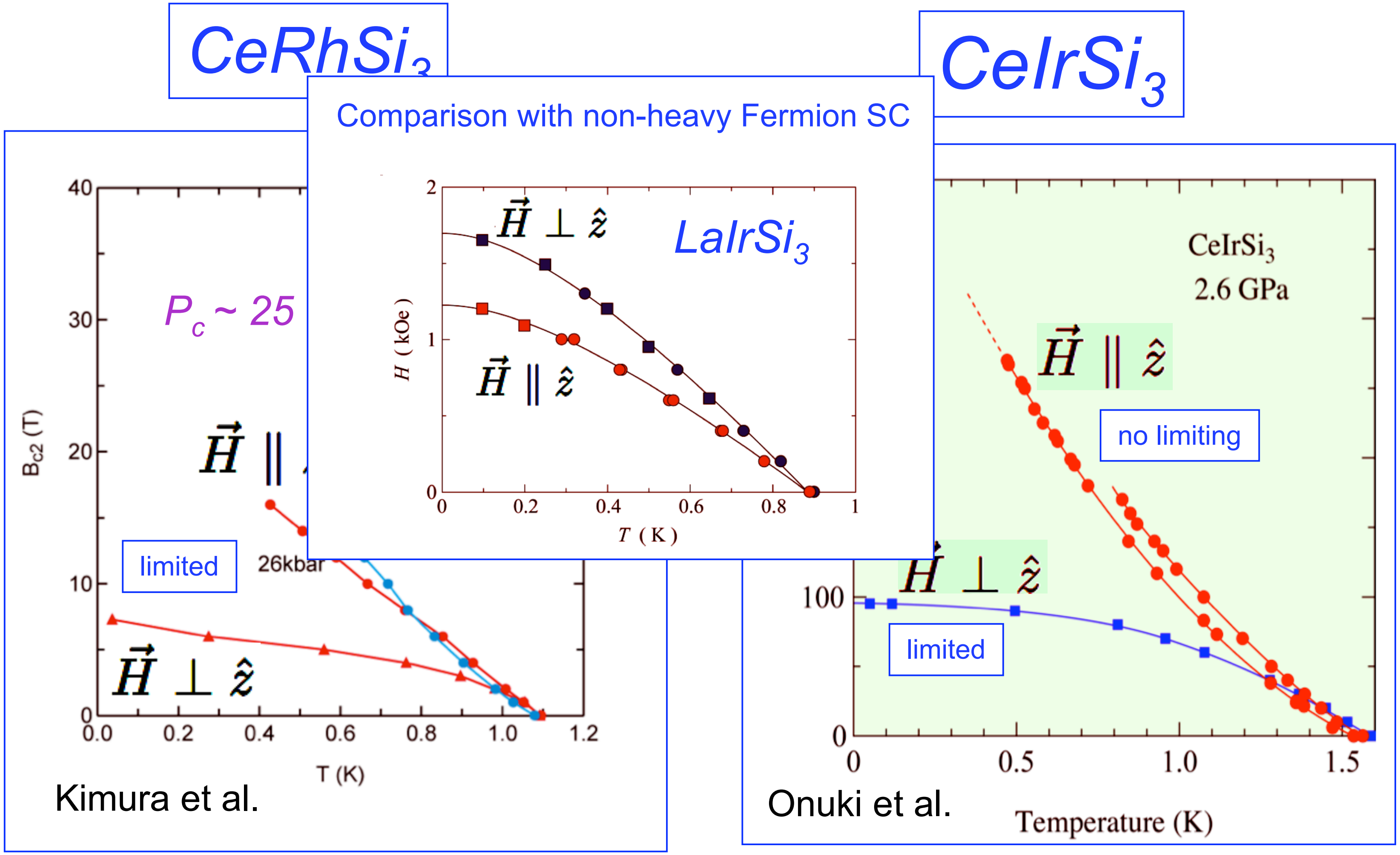
$\vec{H} \parallel \hat{z}$ no limiting

$\vec{H} \perp \hat{z}$ limited

Onuki et al.

fits very well to theoretical expectations of paramagnetic limiting

Upper critical field and paramagnetic limiting



$CeRhSi_3$

Comparison with non-heavy Fermion SC

$CeIrSi_3$

Kimura et al.

Onuki et al.

fits very well to theoretical
expectations of paramagnetic limiting

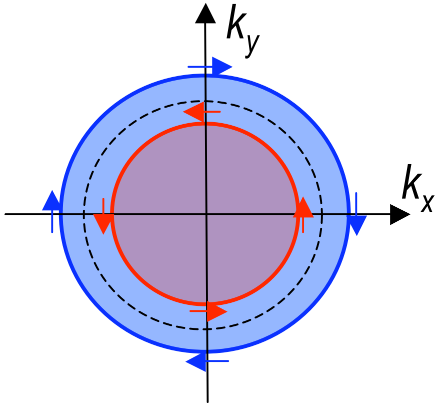
Special features of mixed-parity states

- intrinsic multi-band aspect
- twin boundaries
- surface states

Structure of the pair wave function

$$\hat{\Psi}(\vec{k}) = \left\{ \Delta_1 \psi(\vec{k}) + \Delta_2 \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right\} i\hat{\sigma}^y$$

2 Fermi surfaces $\xi_{\vec{k}\pm} = \xi_{\vec{k}} \pm \alpha \left| \vec{k} \times \hat{z} \right|$

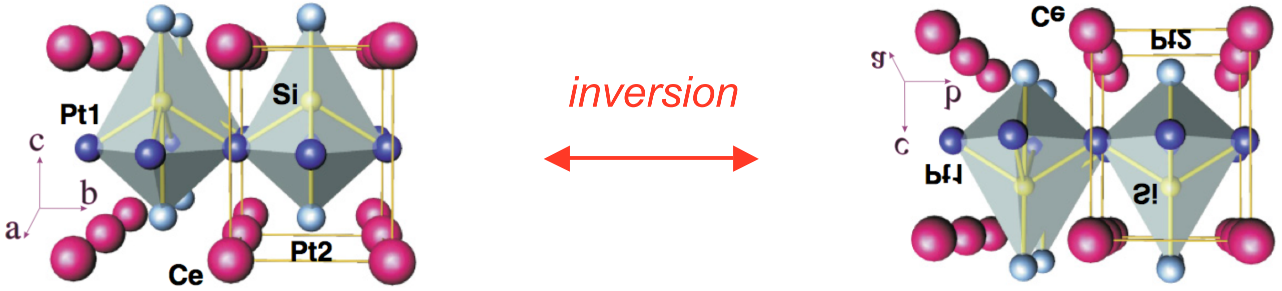


2 different pair wave functions:

$$\Psi_\pm(\vec{k}) = \Delta_1 \psi(\vec{k}) \pm \Delta_2 \frac{\vec{d}(\vec{k}) \cdot \vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

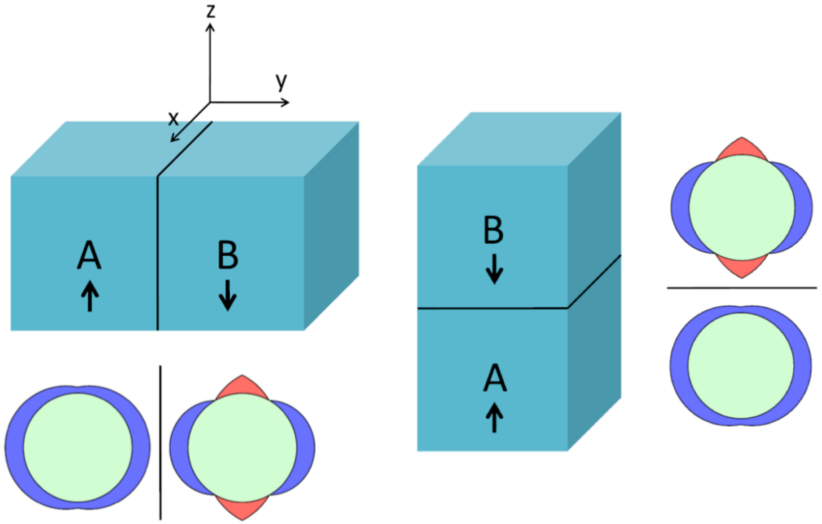
States at twin boundaries

non-centrosymmetric crystals can be twinned



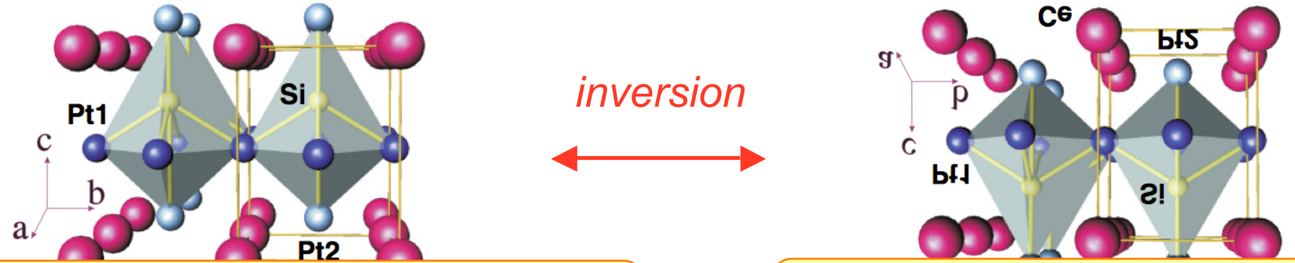
Twin boundaries:

Fermi surfaces exchange role



States at twin boundaries

non-centrosymmetric crystals can be twinned



$$\Psi_A(\vec{k}) = \left\{ \Delta_1 \psi(\vec{k}) + \Delta_2 \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right\} i\hat{\sigma}^y$$

$$\Psi_B(\vec{k}) = \left\{ \Delta_1 \psi(\vec{k}) - \Delta_2 \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}} \right\} i\hat{\sigma}^y$$

Twin boundaries:

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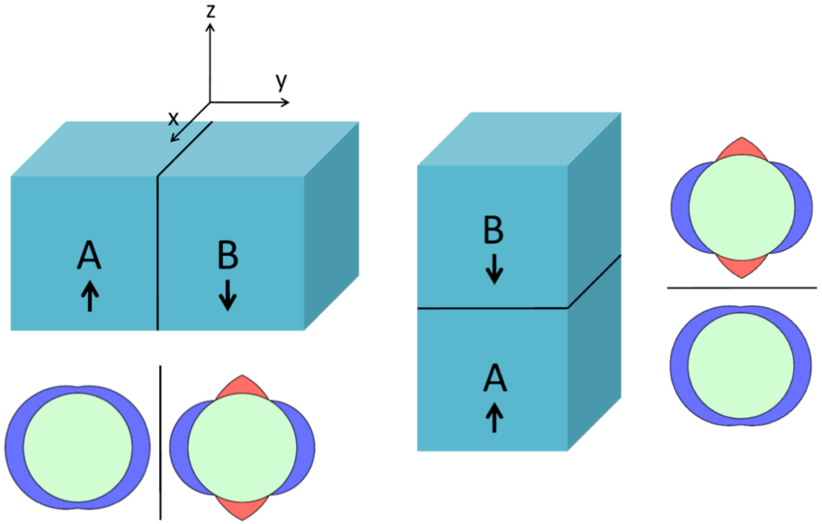
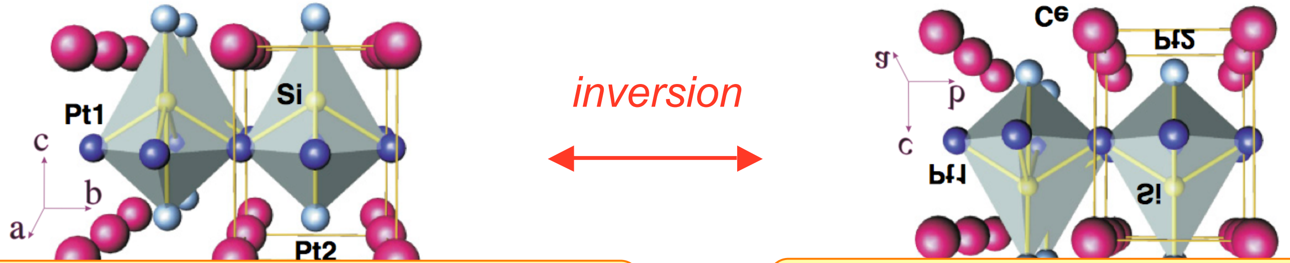


States at twin boundaries

non-centrosymmetric crystals can be twinned

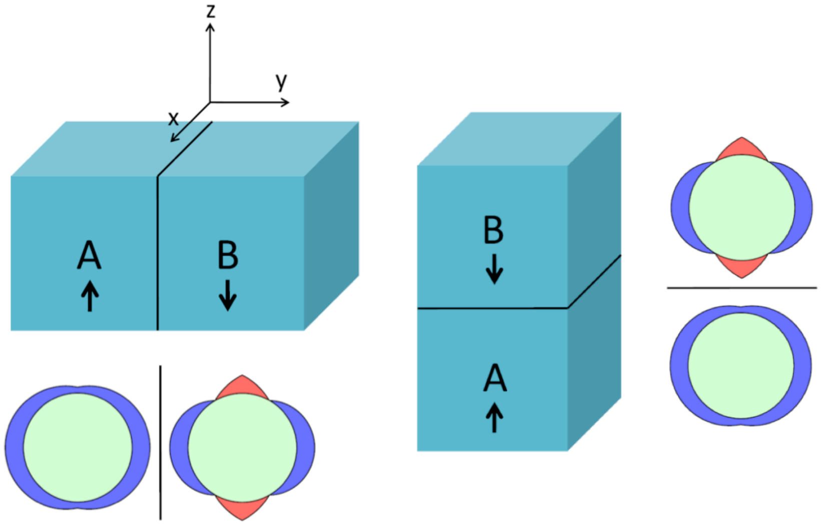


$$\Psi_A(\vec{k}) = \left\{\Delta_1\psi(\vec{k}) + \Delta_2\vec{d}(\vec{k})\cdot\hat{\vec{\sigma}}\right\} i\hat{\sigma}^y$$

$$\Psi_B(\vec{k}) = \left\{\Delta_1\psi(\vec{k}) - \Delta_2\vec{d}(\vec{k})\cdot\hat{\vec{\sigma}}\right\} i\hat{\sigma}^y$$

Twin boundaries:

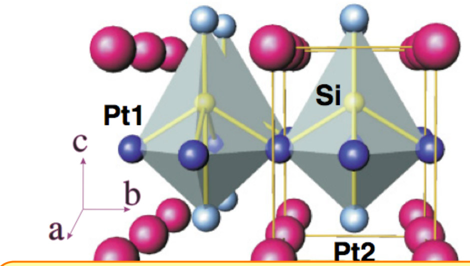
Fermi surfaces exchange role



$$\left.\begin{matrix} e^{i\phi_B}(|\Delta_1|, -|\Delta_2|) \\ e^{i\phi_A}(|\Delta_1|, |\Delta_2|) \end{matrix}\right\} \boxed{\phi = \phi_A - \phi_B}$$

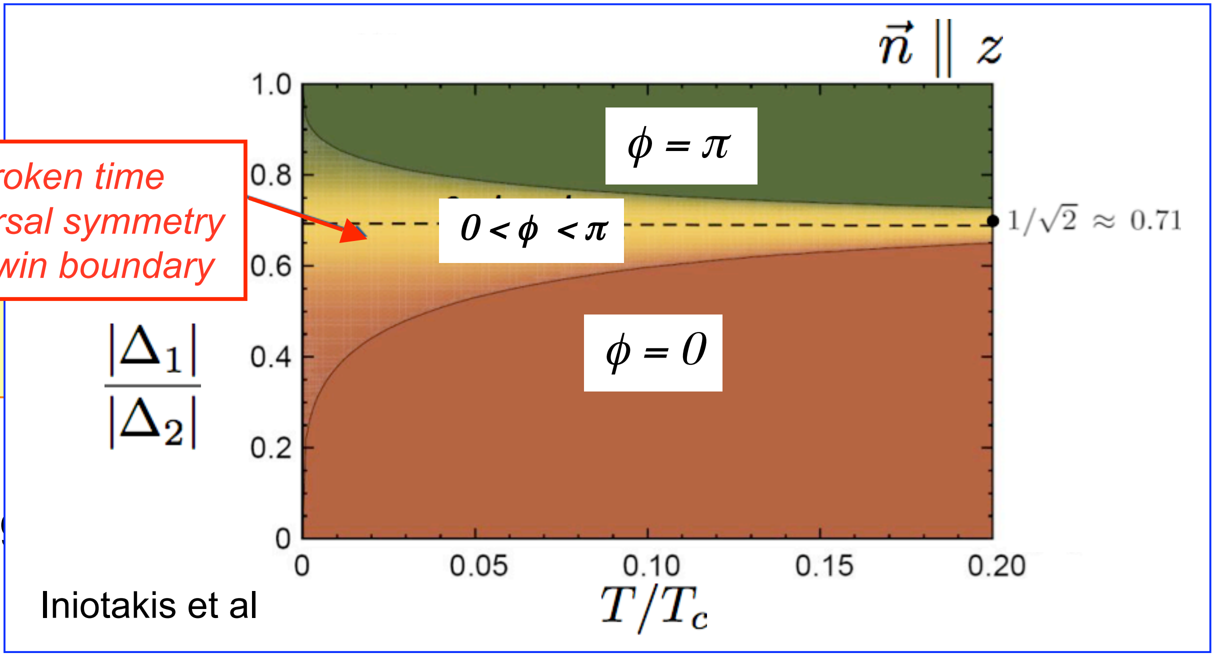
States at twin boundaries

non-centrosymmetric crystals can be twinned



$$\Psi_A(\vec{k}) = \left\{\Delta_1 \psi(\vec{k}) + \Delta_2 \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\right\}$$

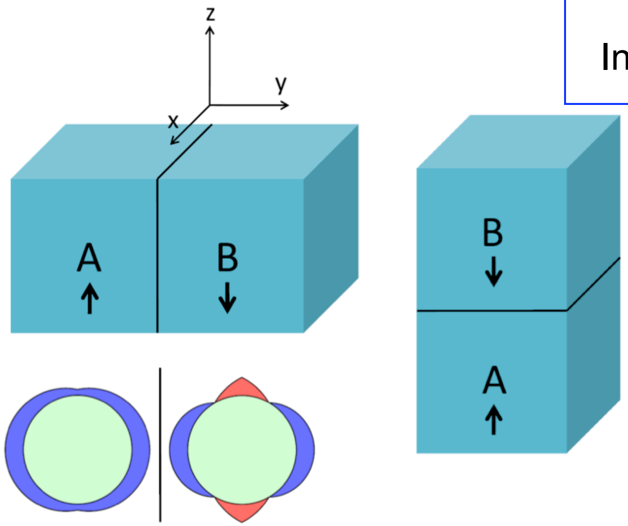
broken time reversal symmetry on twin boundary

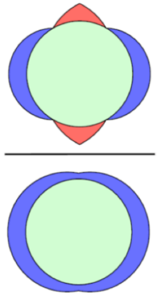


Iniotakis et al

Twin boundaries:

Fermi surfaces exchan



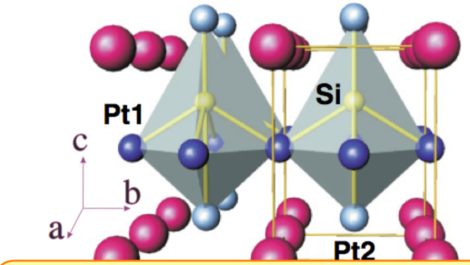


$$\left.\begin{array}{l} e^{i\phi_B}(|\Delta_1|, -|\Delta_2|) \\ e^{i\phi_A}(|\Delta_1|, |\Delta_2|) \end{array}\right\}$$

$$\phi = \phi_A - \phi_B$$

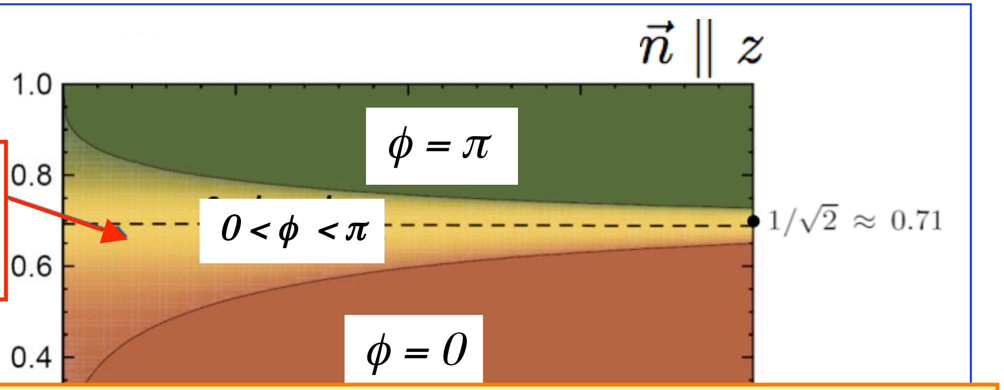
States at twin boundaries

non-centrosymmetric crystals can be twinned



$$\Psi_A(\vec{k}) = \left\{\Delta_1 \psi(\vec{k}) + \Delta_2 \vec{d}(\vec{k}) \cdot \hat{\vec{\sigma}}\right\}$$

broken time reversal symmetry on twin boundary



broken time reversal symmetry

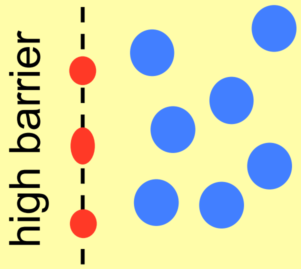
line defects of phase ϕ on twin boundary

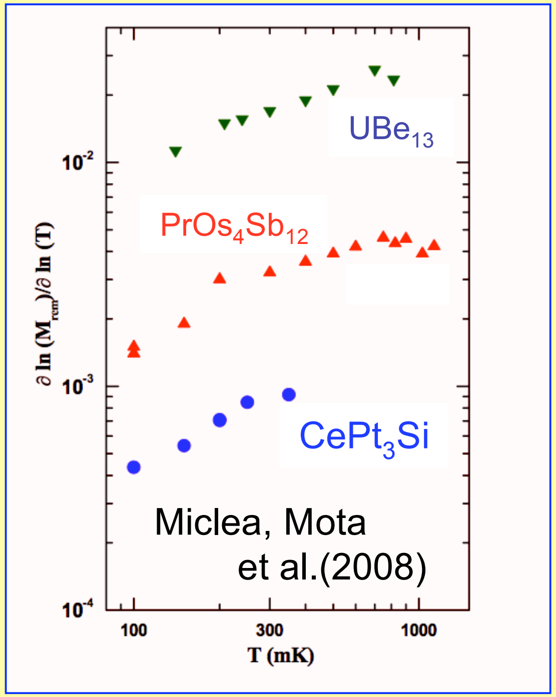
⬇

fractional vortices strongly pinned on twin boundary

Iniotakis, Fujimoto, Savary & MS

impediment for flux flow

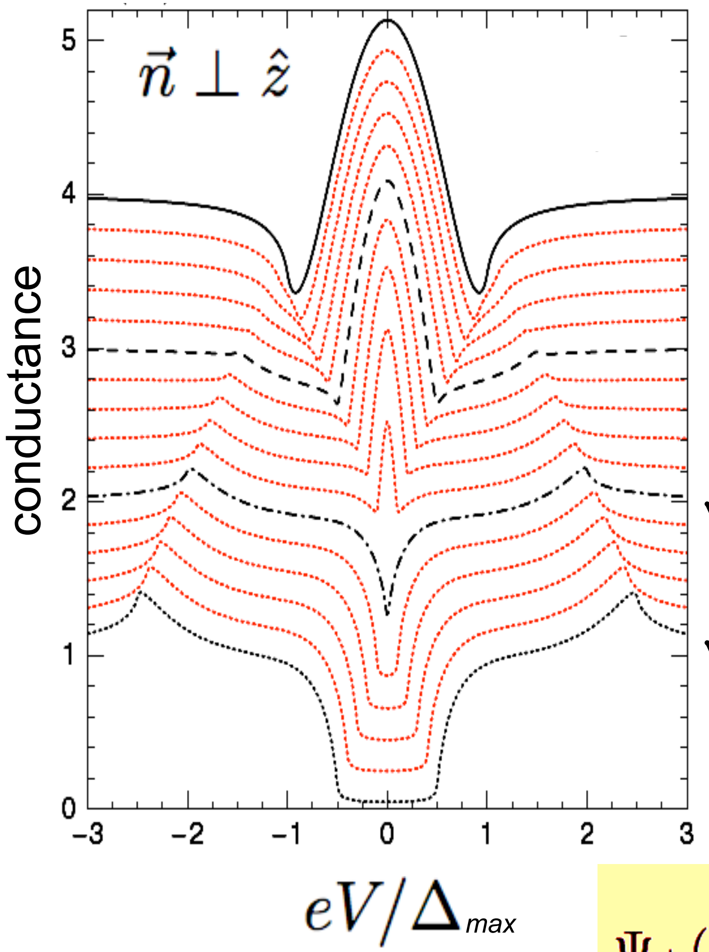




Miclea, Mota et al.(2008)

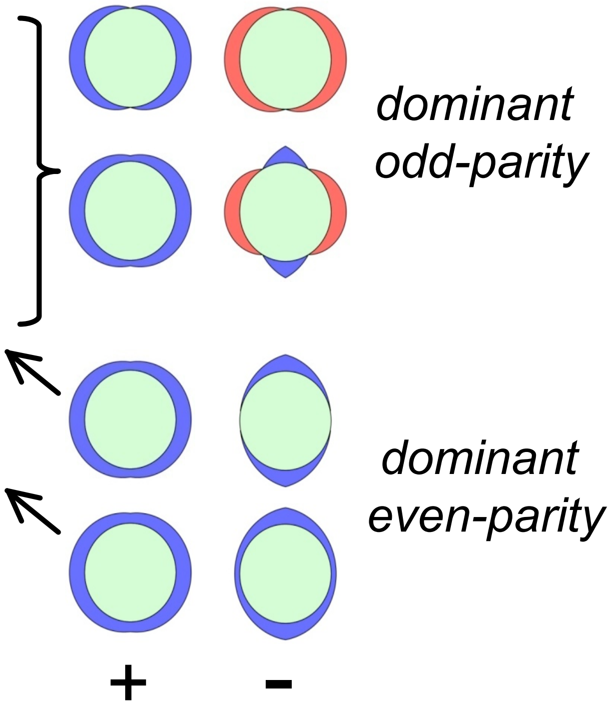
Quasiparticle tunneling

multigap features and zero-bias anomalies



ratio
s- vs *p*-wave
even vs odd

$|\Delta_1|/|\Delta_2|$



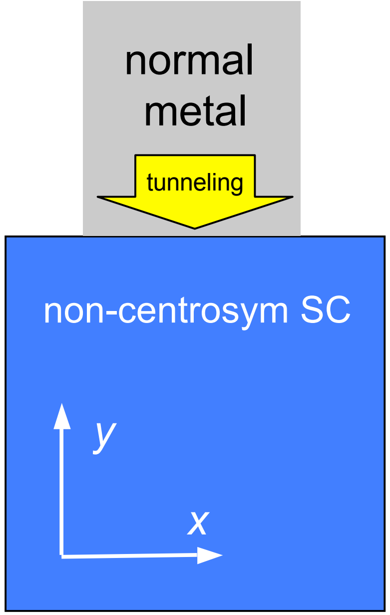
dominant odd-parity

dominant even-parity

\+ −

$$\Psi_{\pm}(\vec{k}) = \Delta_1 \psi(\vec{k}) \pm \Delta_2 \frac{\vec{d}(\vec{k}) \cdot \vec{\lambda}_{\vec{k}}}{|\vec{\lambda}_{\vec{k}}|}$$

NS-tunneling spectroscopy



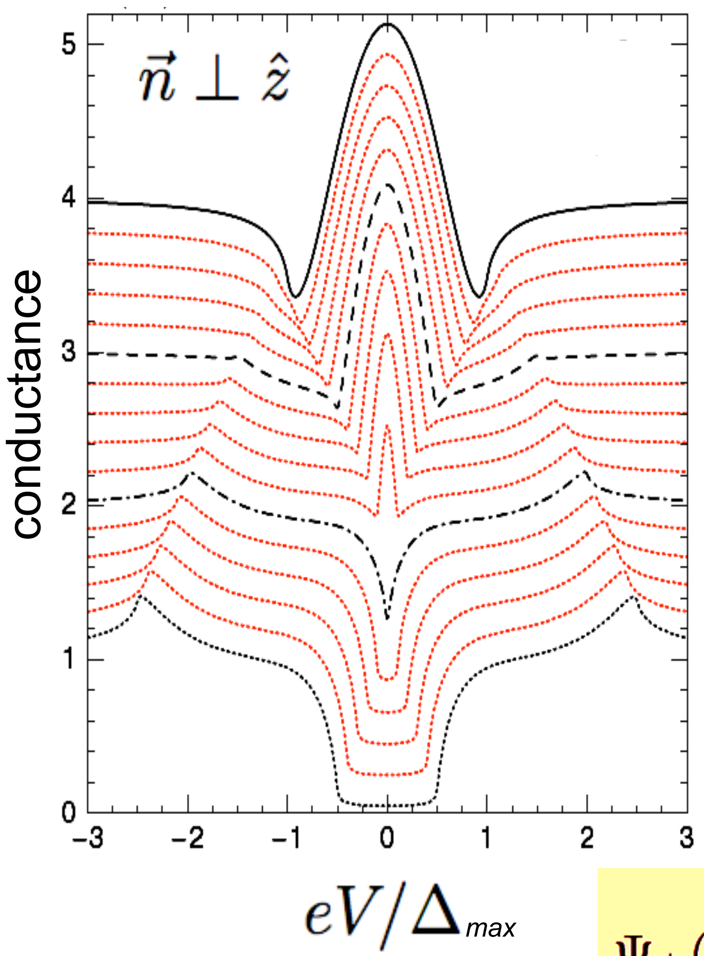
normal metal

tunneling

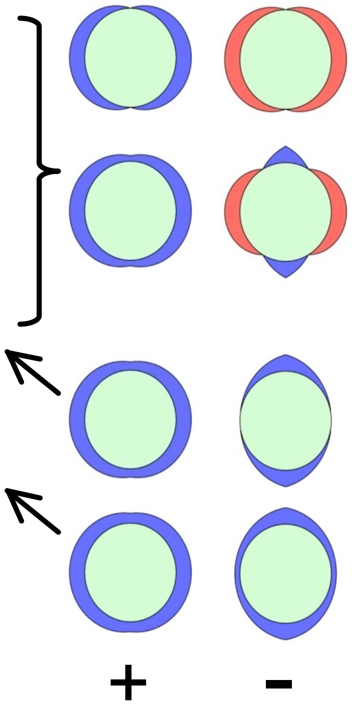
non-centrosym SC

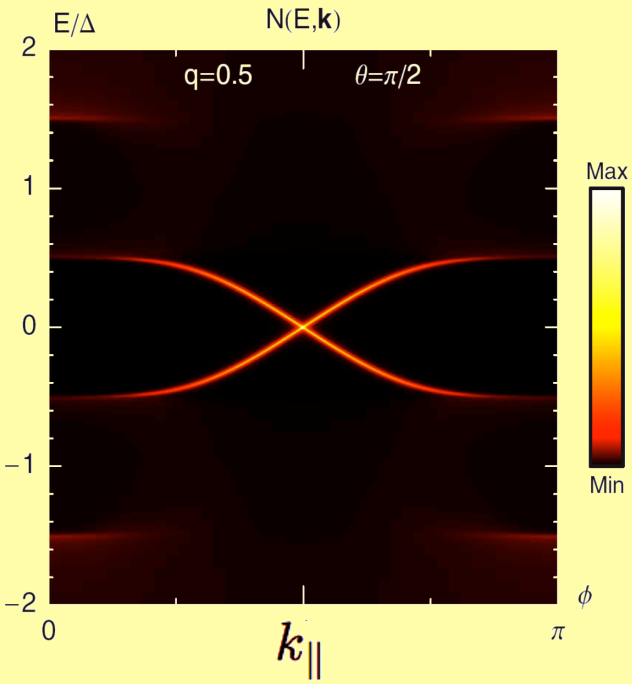
Quasiparticle tunneling

multigap features and zero-bias anomalies



ratio
s- vs *p*-wave
even vs odd





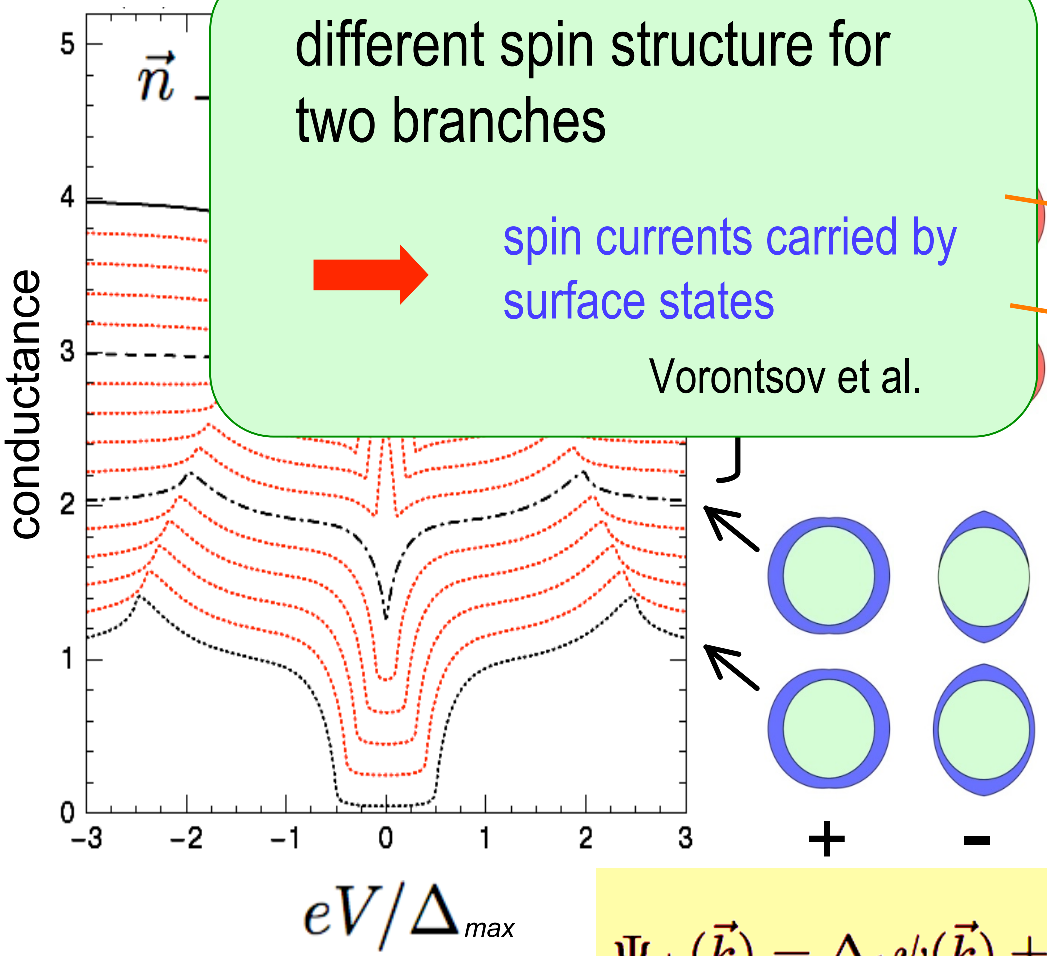
surface bound states
for dominantly
odd-parity pairing state

C. Iniotakis et al. (2007)

$$\Psi_{\pm}(\vec{k}) = \Delta_1 \psi(\vec{k}) \pm$$

Quasiparticle tunneling

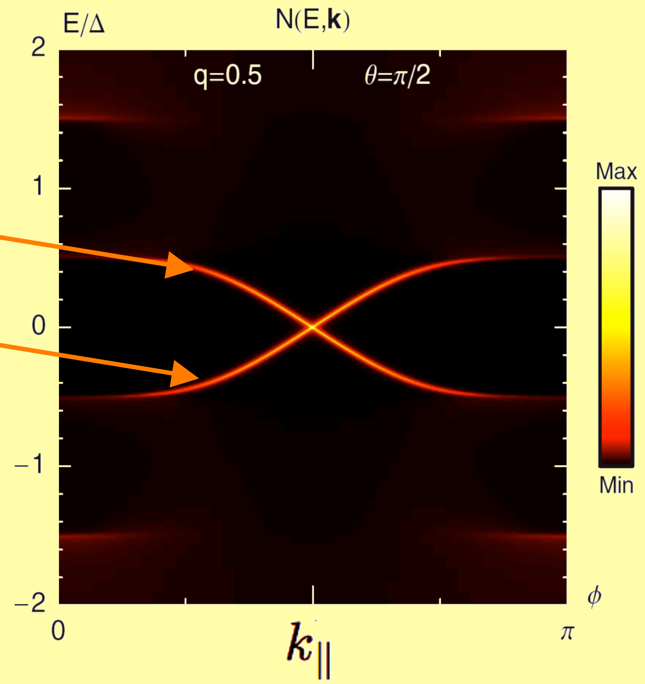
multigap features and zero-bias anomalies



different spin structure for two branches

➡ spin currents carried by surface states

Vorontsov et al.



surface bound states
for dominantly
odd-parity pairing state

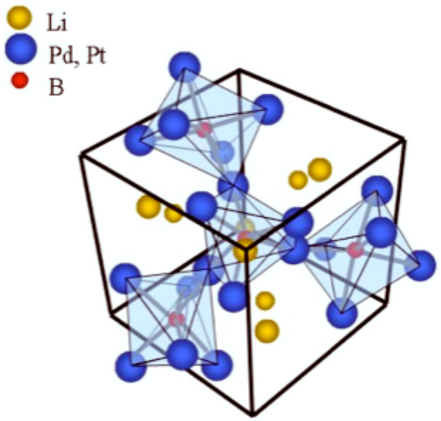
C. Iniotakis et al. (2007)

$$\Psi_{\pm}(\vec{k}) = \Delta_1 \psi(\vec{k}) \pm$$

Li_2Pd_3B & Li_2Pt_3B

Unequal twins

Li_2Pd_3B, Li_2Pt_3B



Togano et al. (2004)

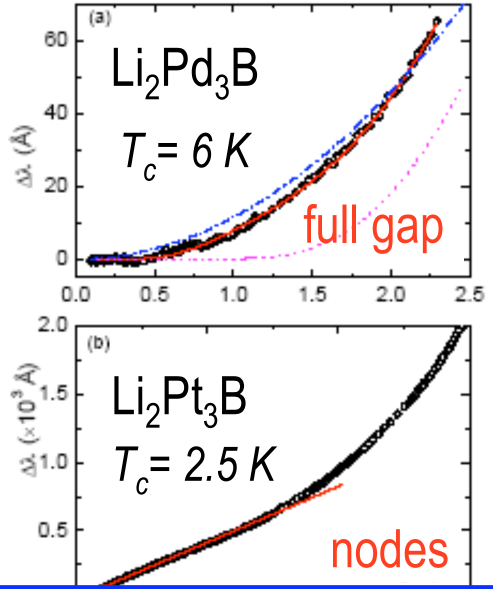
Space group:

$P4_332$ cubic

alloy interpolation:

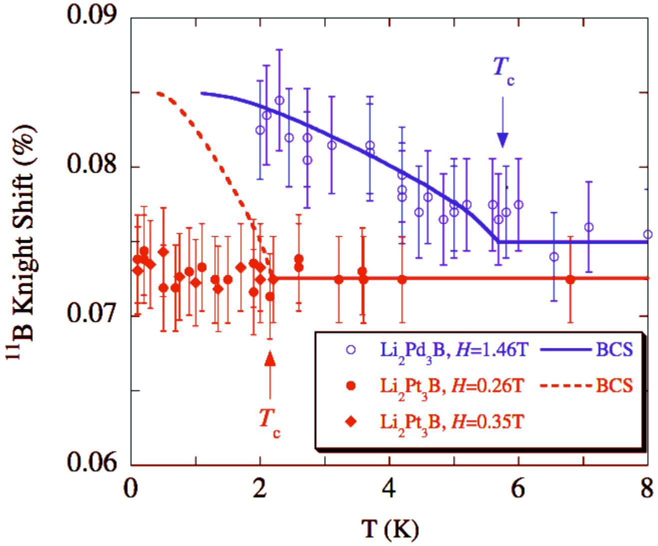
$Li_2(Pd_xPt_{1-x})_3B$

London penetration depth



Yuan et al. (2005)

Nuclear magnetic resonance



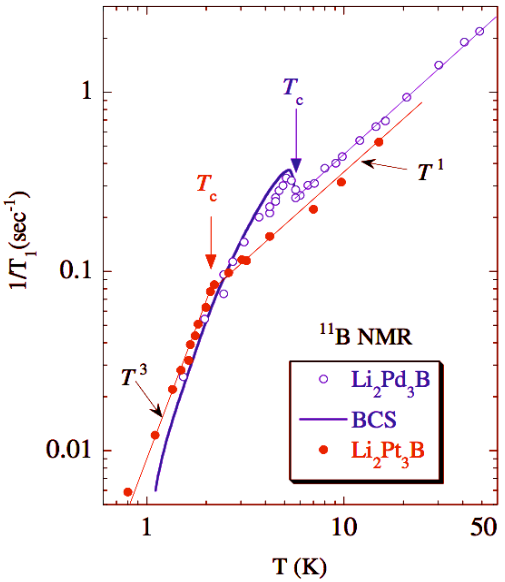
Li_2Pd_3B

conventional

Li_2Pt_3B

unconventional

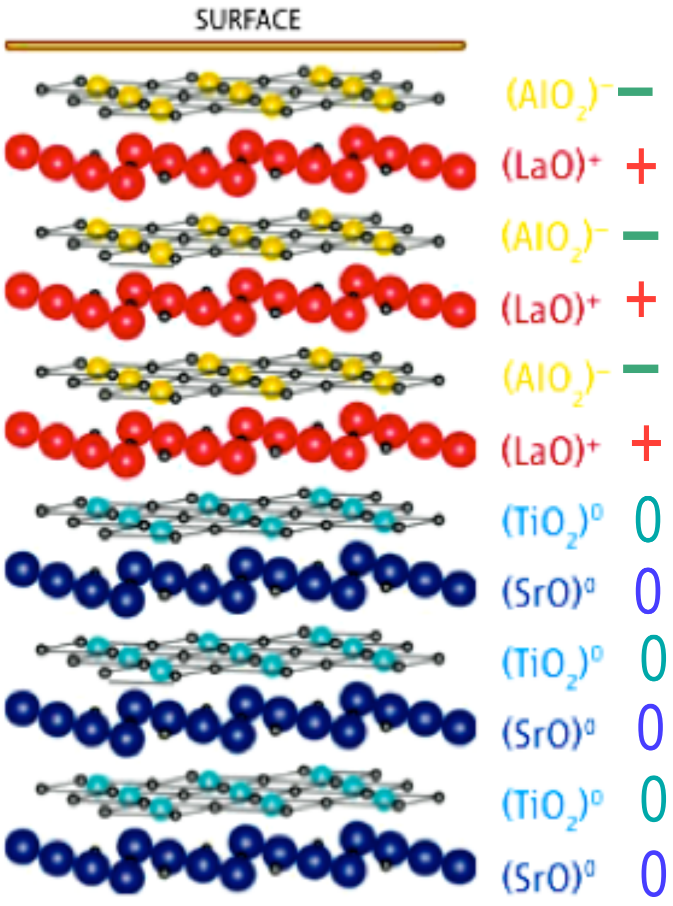
Zheng et al. (2007)



$LaAlO_3$ / $SrTiO_3$ Heterostructure

Environment for non-centrosymmetric superconductivity

$LaAlO_3$ / $SrTiO_3$ heterostructure



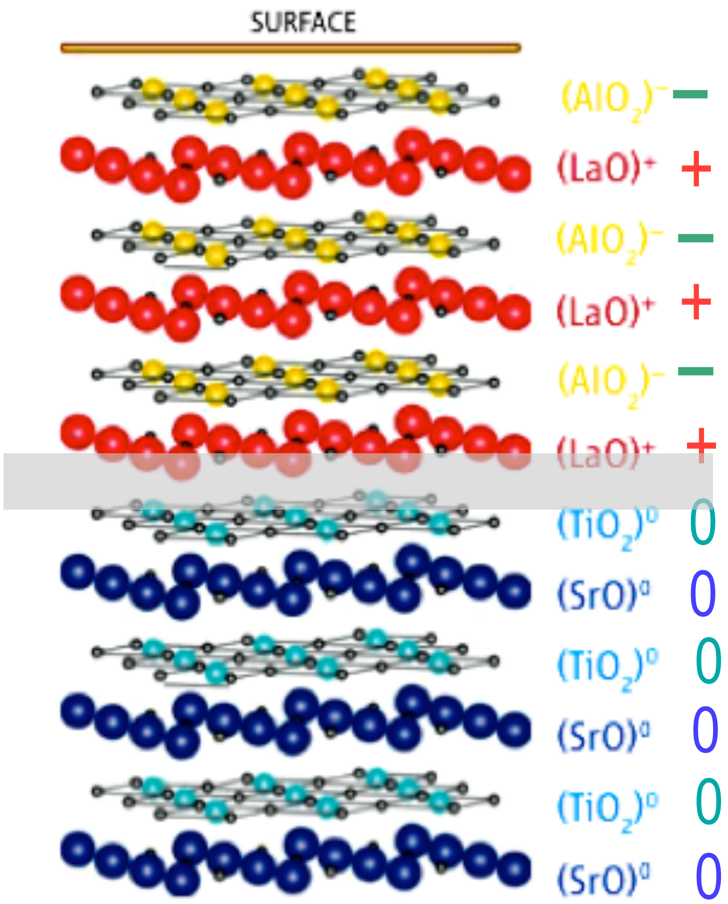
polar

band insulator

non-polar

band insulator

$LaAlO_3$ / $SrTiO_3$ heterostructure

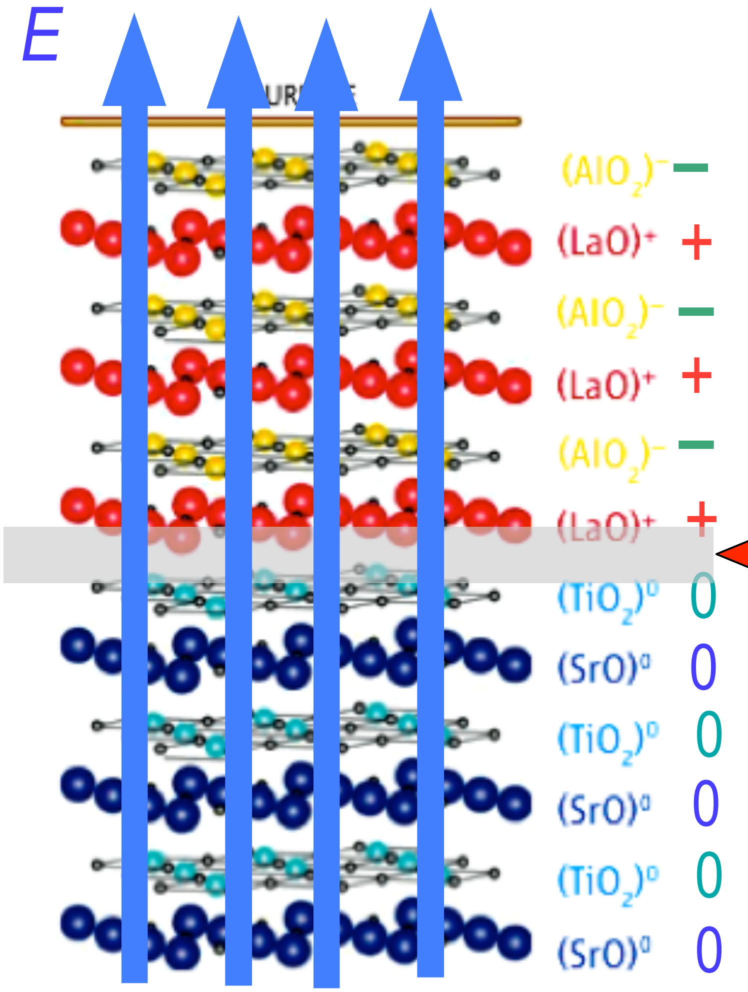


polar
band insulator

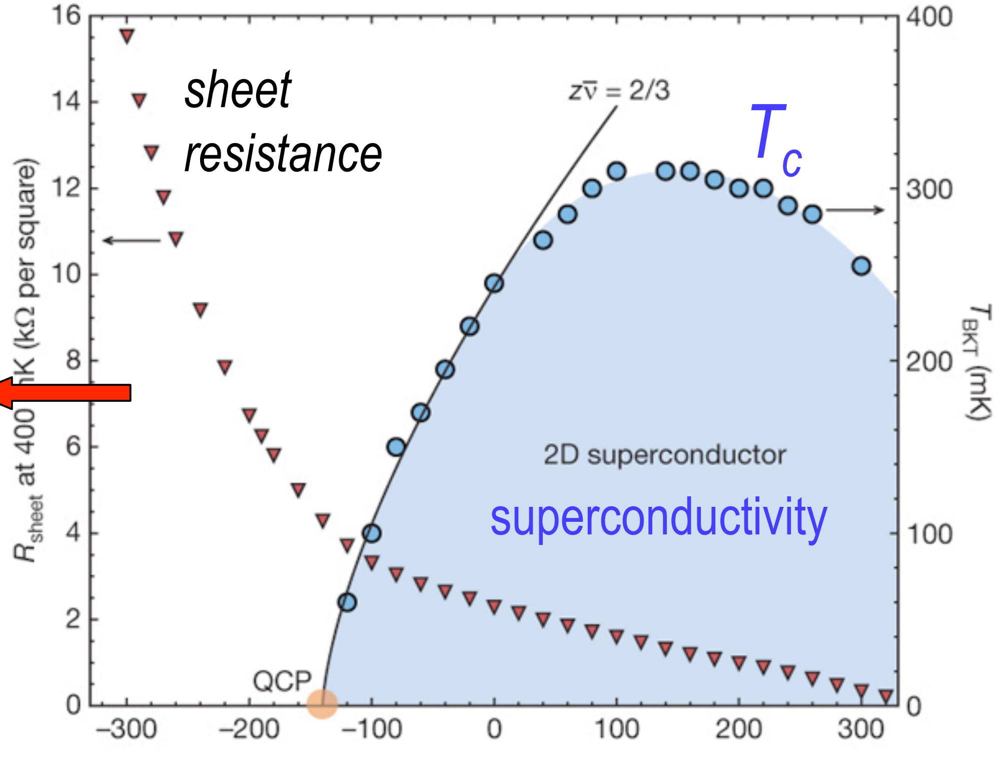
electronic
reconstruction
metallic layer

non-polar
band insulator

$LaAlO_3$ / $SrTiO_3$ heterostructure



interface superconductivity



sheet resistance

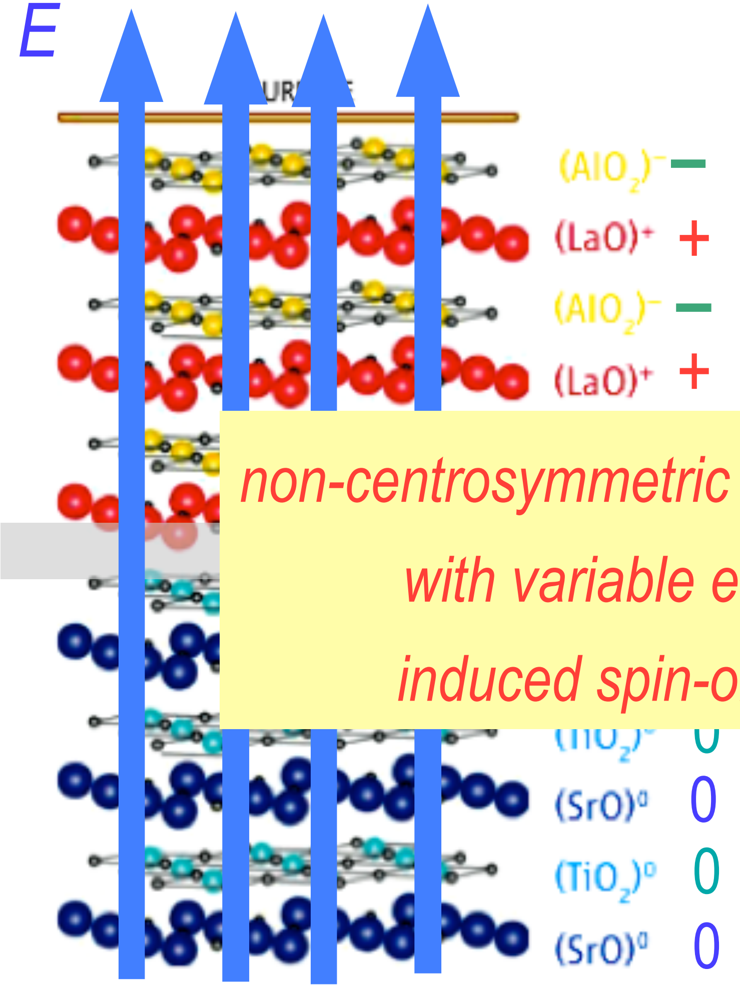
T_c

superconductivity

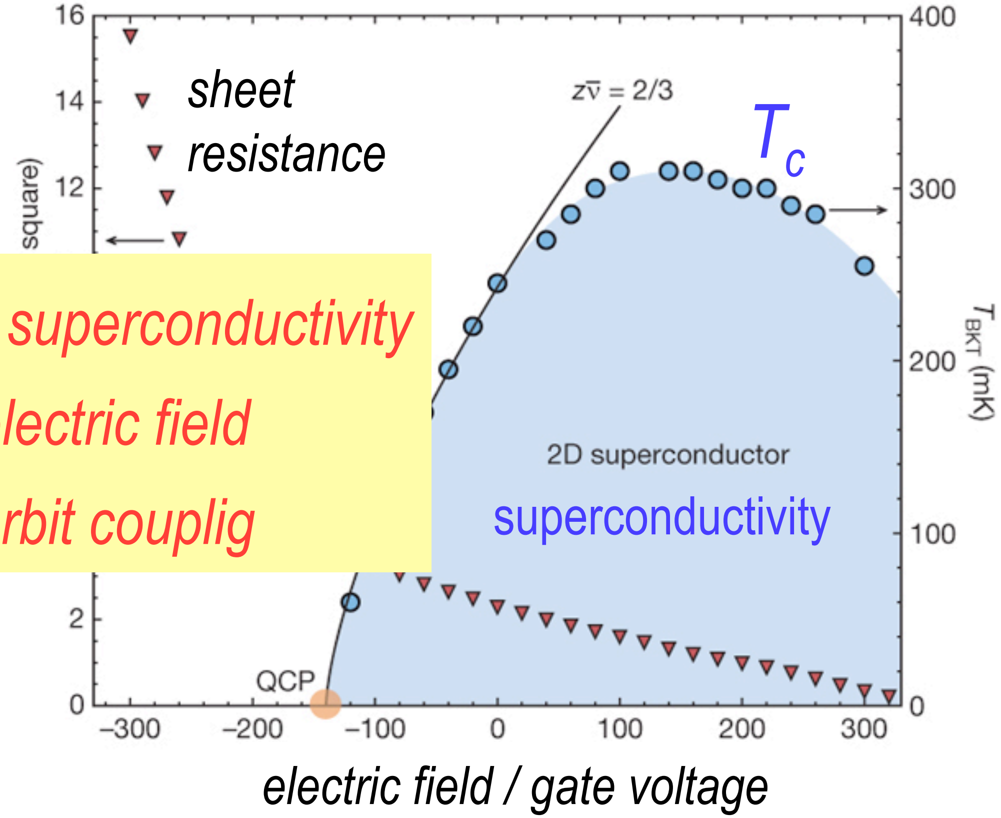
electric field / gate voltage

Reyren, Triscone, Mannhart et al. (2008)

LaAlO$_3$ / SrTiO$_3$ heterostructure



interface superconductivity



non-centrosymmetric superconductivity

with variable electric field

induced spin-orbit couplig

Reyren, Triscone, Mannhart et al. (2008)

Conclusions

- Cooper Pairing involves two key symmetries $\left\{ \begin{array}{l} \textit{time reversal} \\ \textit{inversion} \end{array} \right.$

- non-unitary states: spin polarized pairing lack of time reversal
  mixed-parity pairing lack of inversion

- non-centrosymmetric superconductors with unconventional pairing

⇨ rich in phenomena with a complex phenomenology

magnetoelectric phenomena connection to spintronics and multiferroics

Edelstein, Mineev, Samokhin, Eschrig, …

Josephson effect phase sensitive probes

Hayashi, Linder, Subdo, Borkje, …

Coexistence of magnetism and superconductivity at quantum critical points

Yanase, Fujimoto, …

Collaborators:

Theory

ETH Zurich: P.A. Frigeri, N. Hayashi, K. Wakabayashi, I. Milat, Y. Yanase
C. Iniotakis, D. Perez, L. Savary, T. Neupert, …
Uni Wisconsin: D.F. Agterberg, R.P. Kaur
Japan: A. Koga, Y. Tanaka, S. Fujimoto, ….

Experiment

TU Wien: E. Bauer and team
Japan: T. Shibauchi, Y. Matsuda, N. Kimura, Y. Onuki, Z. Hiroi, ….
IBM Watson Lab: L. Kruzin-Elbaum
Uni Illinois UC: H. Yuan, M. Salamon and team
Venecuela: I. Bonalde
Geneva & Augsburg: J.M. Triscone, J. Mannhart and teams

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Schweizerischer Nationalfonds
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Swiss National Science Foundation

MaNEP
SWITZERLAND

TOYOTA CENTRAL R&D LABS., INC.