Extra dimensions and a mass hierarchy of elementary particles

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"Weaving Science Web beyond Particle-Matter Hierarchy“
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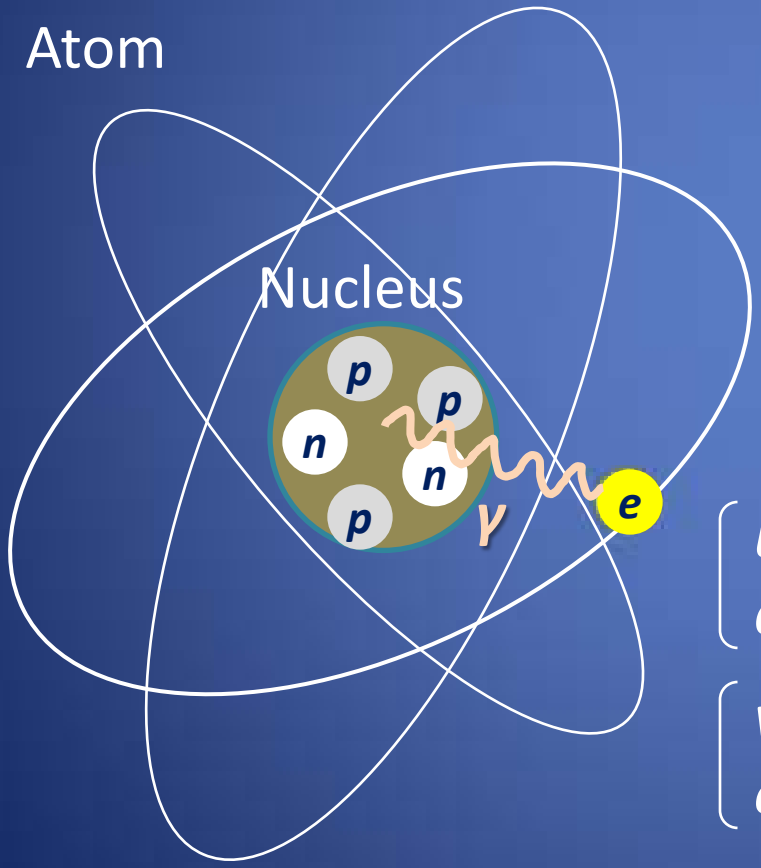
Plan of this talk

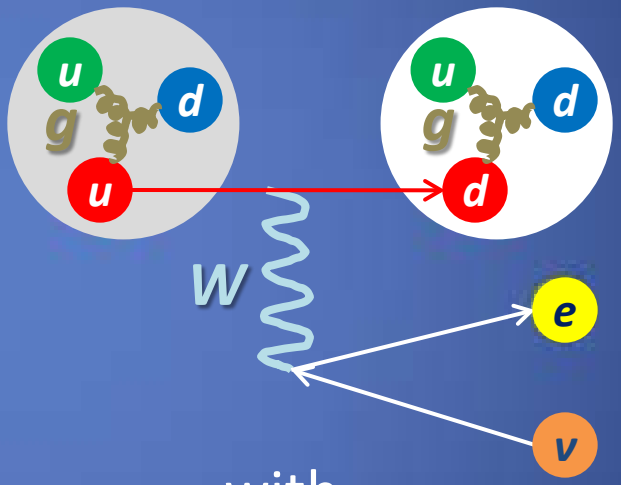
1. Hierarchies among elementary particles
2. Magnetized extra dimensions
3. String models
4. Summary

HIERARCHIES AMONG ELEMENTARY PARTICLES

Elementary particles

What is the most fundamental *elements* of our world?





$\begin{pmatrix} u \\ d \end{pmatrix}$ Quarks

$\begin{pmatrix} \nu \\ e \end{pmatrix}$ Leptons

with

Electromagnetic (γ)

Weak ($W^{\pm}$, Z)

Strong (g)

gauge interactions

The elements of our world

Not the four elements (earth, air, fire, water),
but three generations of quarks & leptons

(Kobayashi-Maskawa '08 Nobel prize)

Quarks $Q_i =$ $\begin{pmatrix} u \\ d \end{pmatrix}$ | $\begin{pmatrix} c \\ s \end{pmatrix}$ | $\begin{pmatrix} t \\ b \end{pmatrix}$

Leptons $L_i =$ $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ | $\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$ | $\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

with
Electromagnetic (γ)
Weak ($W^\pm$, Z)
Strong (g)
gauge interactions
[Gauge theory]

i = 1, 2, 3 distinguished *only by the mass.*

Mass of elementary particles

Masses are restricted by gauge symmetries.

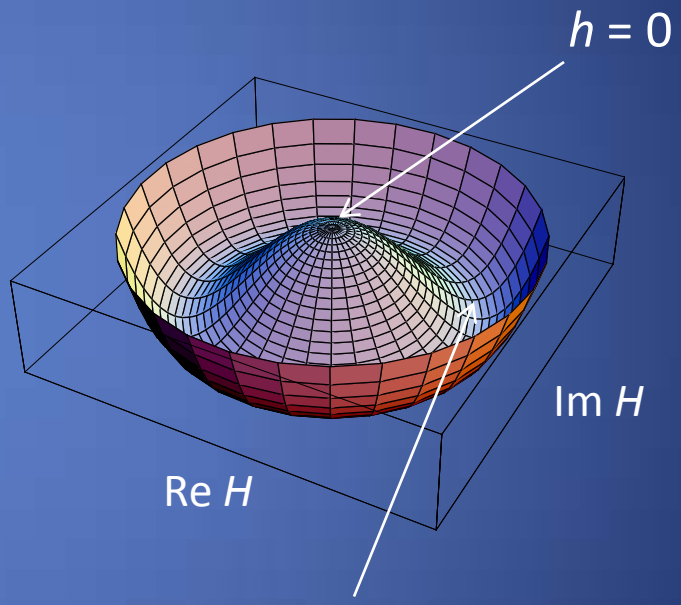
- Spontaneous symmetry breaking is necessary
  (Nambu '08 Nobel prize)

$m_W, m_Z \sim \langle h \rangle \sim 100$ GeV

Scale of weak interaction

[EW standard model]

Higgs field H



$\langle h \rangle \neq 0$

$H = h\, e^{i\varphi}$

Mass of elementary particles

Quark, lepton masses are also generated by $< h >$

$m_i = \eta_i < h > \sim 100$ GeV ?

$\eta_i \sim 1$

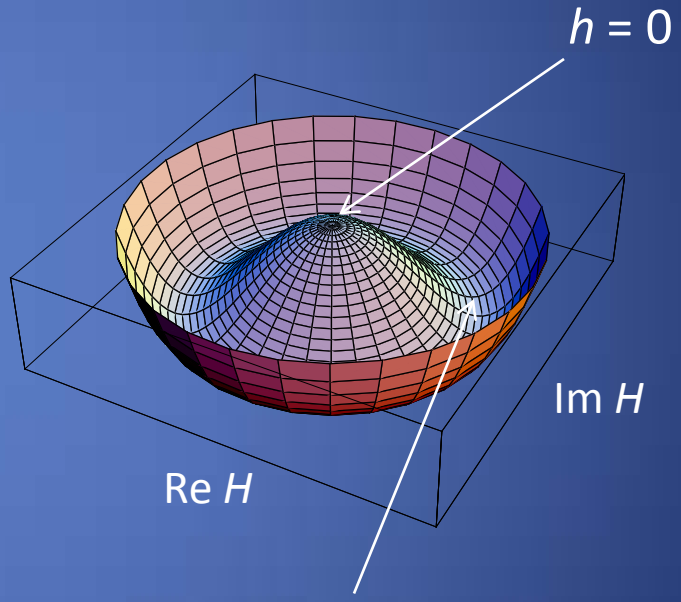
η_i : Eigenvalues of Yukawa matrices

$Y^U_{ij} Q_{Li} U_{Rj} H$, $Y^D_{ij} Q_{Li} D_{Rj} H'$,

$Y^E_{ij} L_{Li} E_{Rj} H'$

(dim-less parameters of the theory)

Higgs field H



$< h > \neq 0$

$H = h\, e^{i\varphi}$

Hierarchy of the elements

Quark, lepton masses are also generated by $< h >$

$m_i = \eta_i < h >$ ⟹

η_i hierarchical

η_i : Eigenvalues of Yukawa matrices

$Y^U_{ij} Q_{Li} U_{Rj} H$, $Y^D_{ij} Q_{Li} D_{Rj} H'$,

$Y^E_{ij} L_{Li} E_{Rj} H'$

(dim-less parameters of the theory)

Experimental values

Q	m_Q (GeV)	L	m_L (GeV)
u	1.5×10^{-3}	ν_L	$\lesssim 10^{-9}$
d	3.5×10^{-3}	e	5.11×10^{-4}
c	1.3	ν_M	$\lesssim 10^{-9}$
s	1.0×10^{-1}	μ	1.06×10^{-1}
t	171.2	ν_H	$\lesssim 10^{-9}$
b	4.2	τ	1.78

Hierarchy of the elements

Qua

g

$$V_{\rm CKM} \equiv V_L^u V_L^{d\dagger} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

m_i =

$$V_{\rm CKM} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

η_i : Eigenvalues of Yukawa matrices

$Y^U_{ij} Q_{Li} U_{Rj} H$, $Y^D_{ij} Q_{Li} D_{Rj} H'$,

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s	1.0×10^{-1}	μ	1.06×10^{-1}
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God put the hierarchy among the elements?

Hints from the unification

- So far, we have neglected gravity.
- Incorporate gravity into elementary particle (quantum) physics

Superstrings

The only known way of

– quantizing gravity without any inconsistencies

– including gauge theories

[Standard model of elementary particles]

The most promising candidate for the theory of everything

Hints from the unification

A prediction of superstring theory

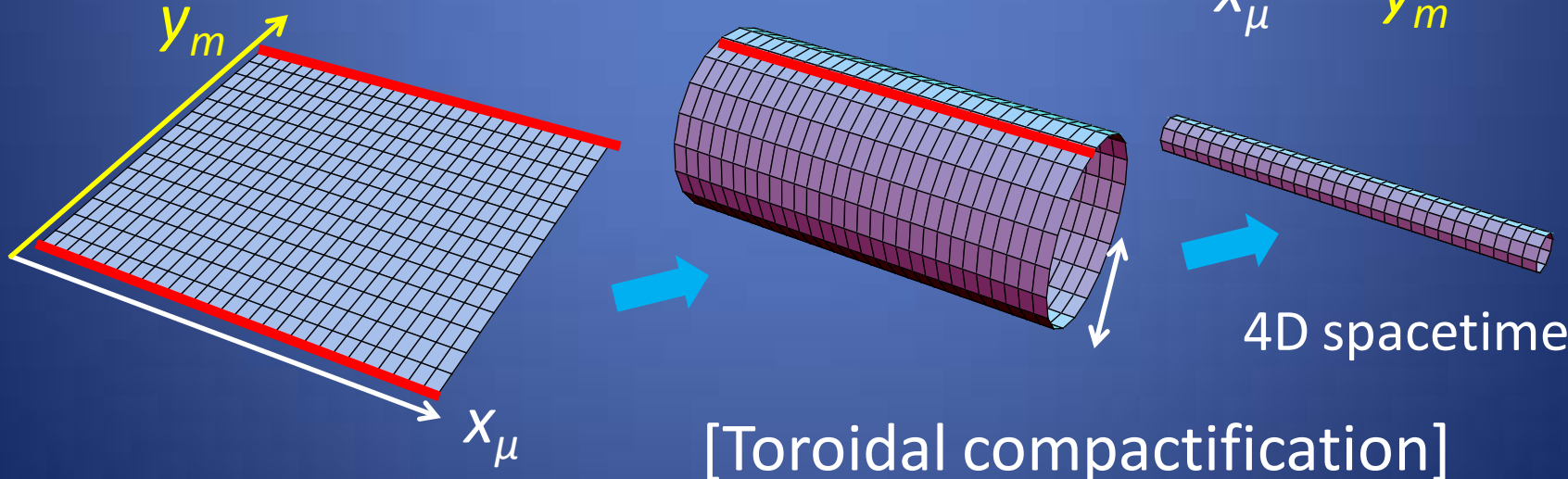
Extra dimensions

Consistencies require

Six extra dims $m = 1, \ldots, 6$

spacetime dimension = 9+1 = (3+1) + 6

x_μ y_m



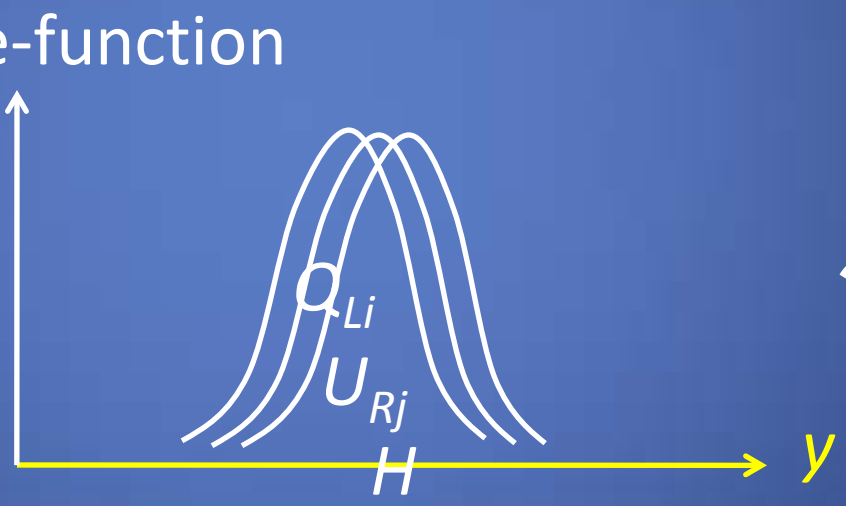
[Toroidal compactification]

Hints from the unification

N. Arkani-Hamed & M. Schmaltz ‘00

Y_{ij} are determined by an overlap integral of wave-functions in extra dims

$$Y_{ij} = \int dy \; \sim 1$$



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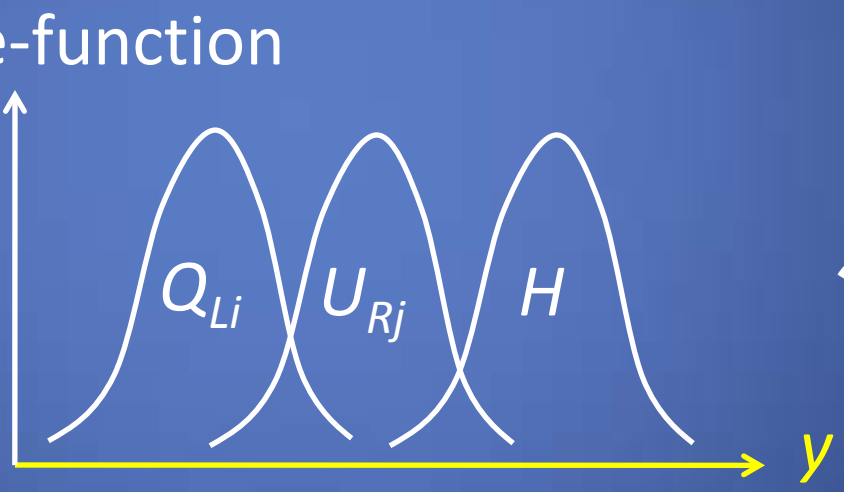


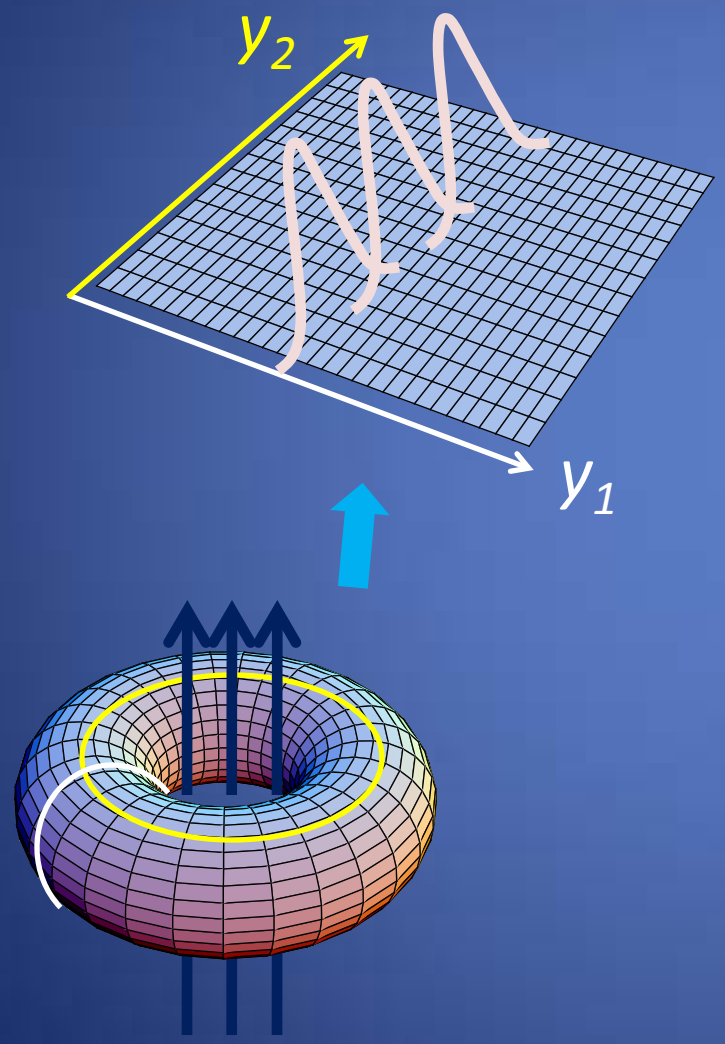
$\sim 10^{-5}$

Nontrivial wave-function profile can be a source of hierarchy in 4D spacetime.

MAGNETIZED EXTRA DIMENSIONS

Magnetic flux in extra dimensions

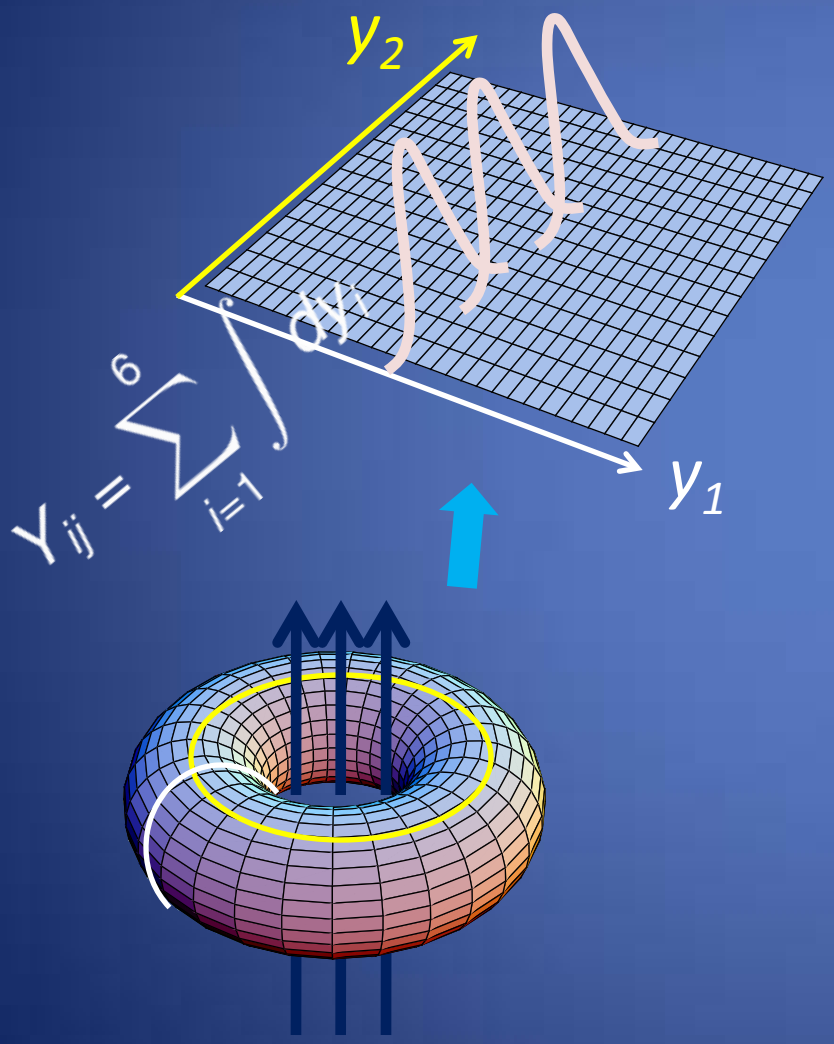
D. Cremades, L.E. Ibanez & F. Marchesano '04



- Charged particles in magnetized extra dims
- Solution of Dirac eq
  - LR asymmetric (chiral)
  - # of flux determines
    # of zero mode
    i.e. # of generation
  - Localized wave-functions
    Jacobi theta function

Hierarchies in 4D

D. Cremades, L.E. Ibanez & F. Marchesano '04

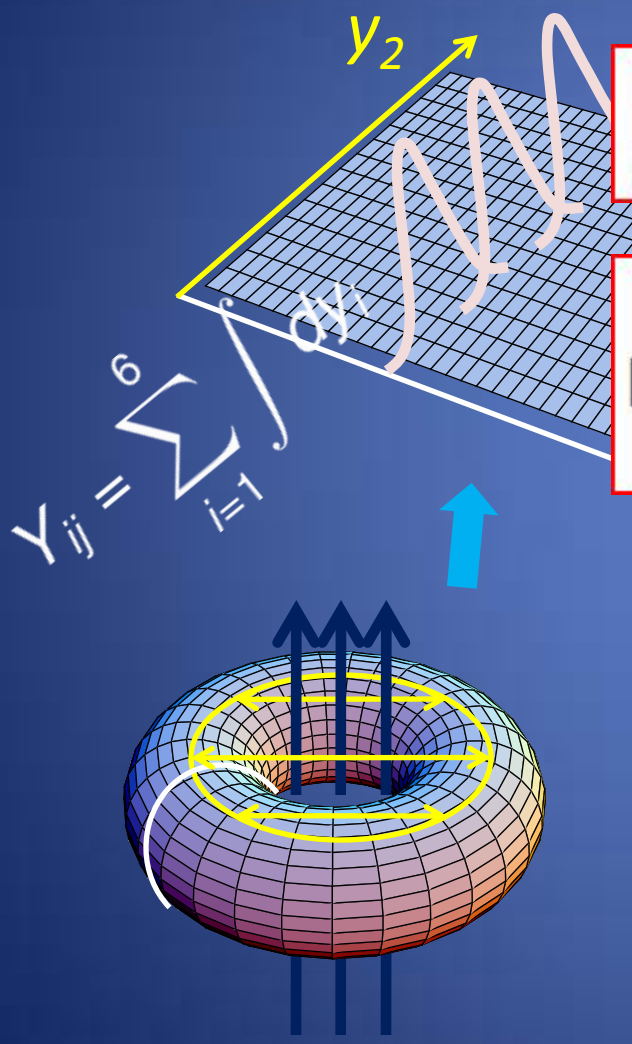


- Overlap integral yields some hierarchical Y_{ij}
  - Governed by Jacobi theta
  - Three-gens, difficult
  - Too many Higgs
  - Not realistic patterns of masses and mixings

Need more structure to be realistic

Magnetized orbifold models

K.S. Choi, T. Kobayashi, H. Ohki & H.A. '08-'09



$$(m_u, m_c, m_t)/m_t \sim (2.9 \times 10^{-5}, 2.5 \times 10^{-2}, 1.0),$$
$$(m_d, m_s, m_b)/m_b \sim (4.4 \times 10^{-3}, 0.18, 1.0),$$

$y_m \to -y_m$

$$|V_{CKM}| \sim \begin{pmatrix} 0.98 & 0.22 & 0.018 \\ 0.22 & 0.98 & 0.0014 \\ 0.017 & 0.0052 & 1.0 \end{pmatrix}$$

- lels
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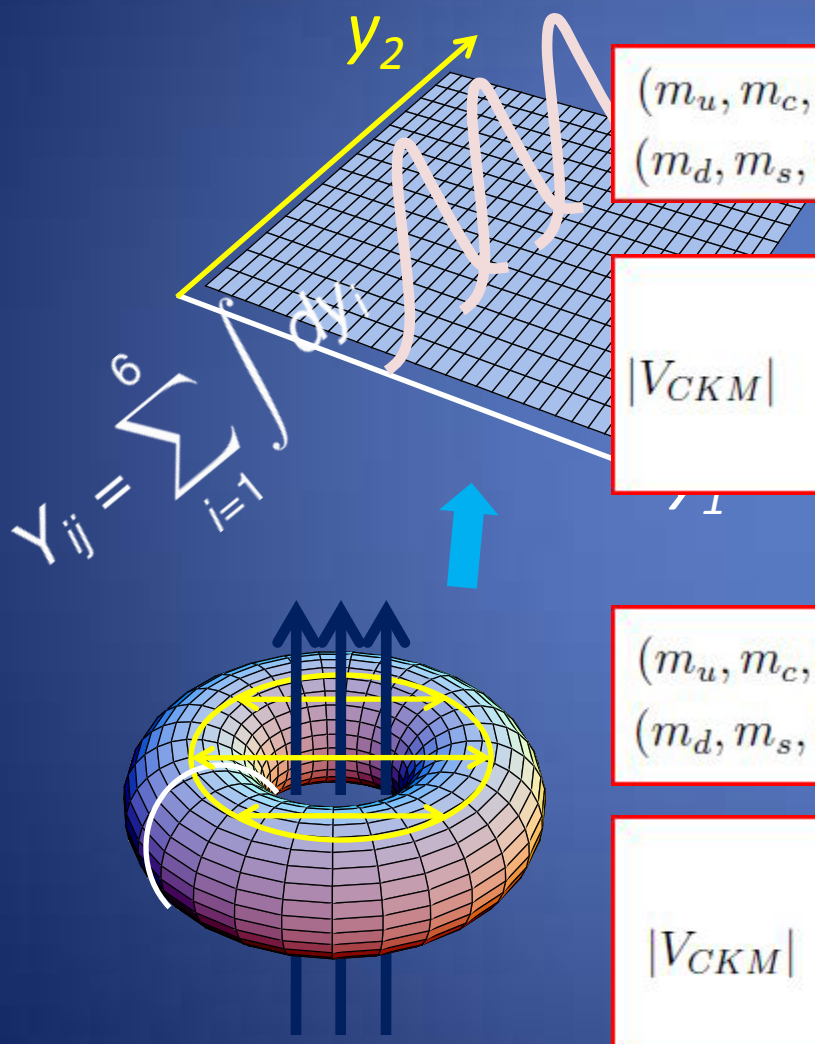
- Higgs
- Semi-realistic (quark) masses and mixings

  [classified 15 patterns]
- Still in progress

Magnetized orbifold models

K.S. Choi, T. Kobayashi, H. Ohki & H.A. '08-'09



$$(m_u, m_c, m_t)/m_t \sim (2.9 \times 10^{-5}, 2.5 \times 10^{-2}, 1.0),$$
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dels

ble]

Higgs

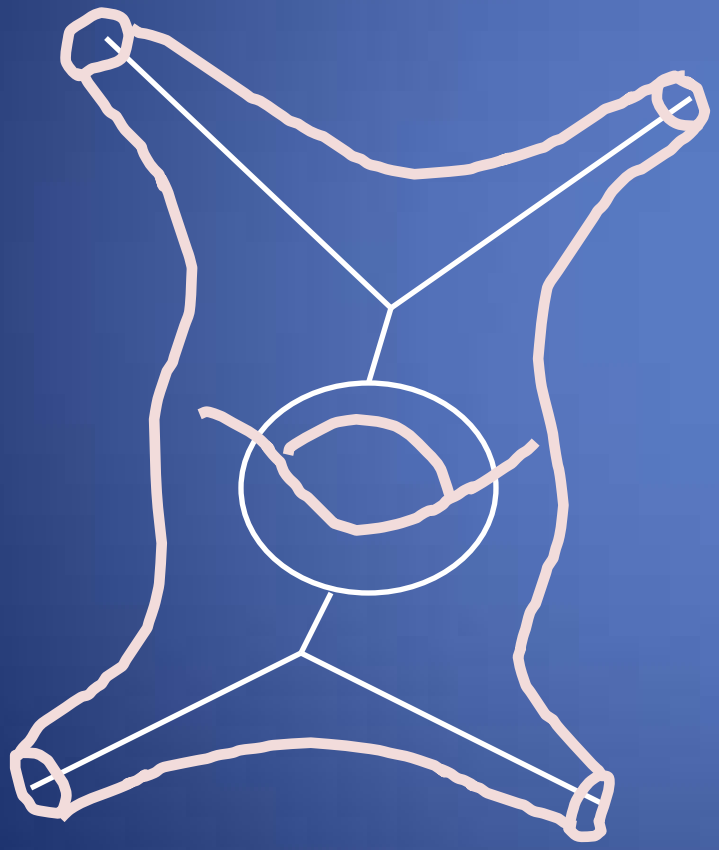
– Semi-realistic (quark)

$$(m_u, m_c, m_t)/m_t \sim (5.6 \times 10^{-6}, 4.7 \times 10^{-3}, 1.0),$$
$$(m_d, m_s, m_b)/m_b \sim (3.3 \times 10^{-3}, 7.1 \times 10^{-2}, 1.0),$$

$$|V_{CKM}| \sim \begin{pmatrix} 0.98 & 0.22 & 0.0034 \\ 0.22 & 0.98 & 0.000081 \\ 0.0033 & 0.00081 & 1.0 \end{pmatrix}$$

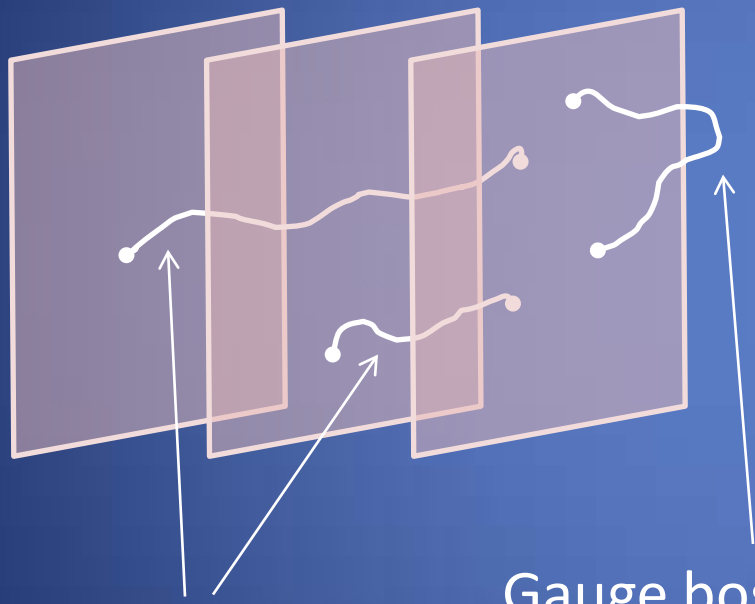
STRING MODELS

Superstrings



- Quantum theory
- Include gravity
  & gauge theories
- No inconsistencies
  - divergences, anomalies
- Only two parameters
  - Tension & Coupling
- Defined perturbatively
  - Many vacua

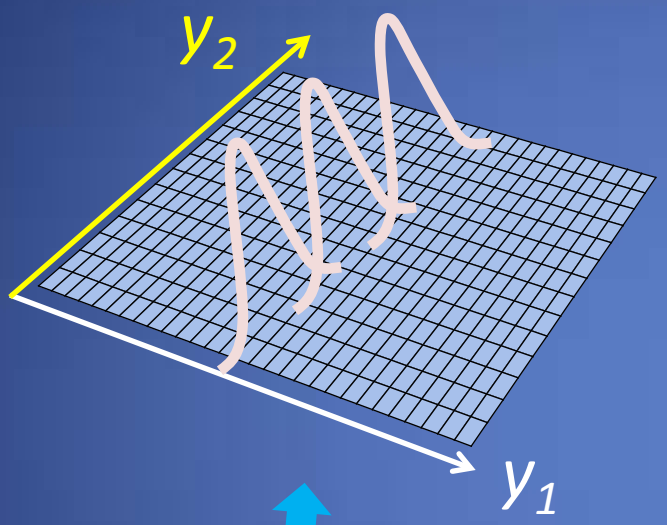
D-branes

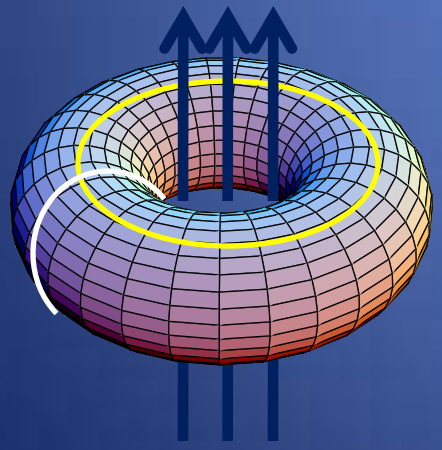


- Superstring theory has a membrane-like soliton

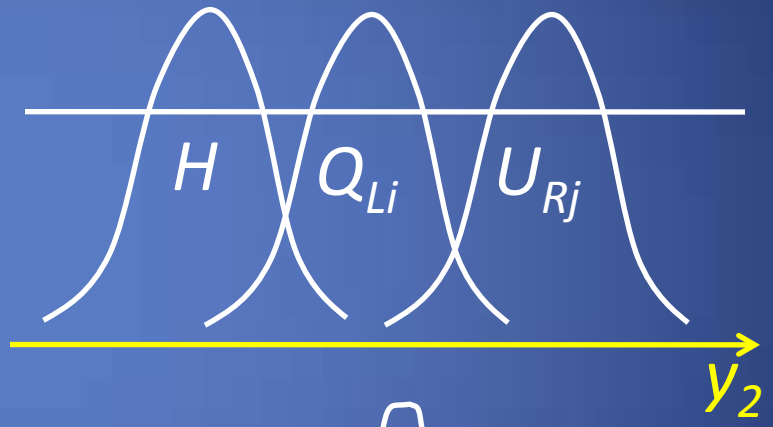
  called `Dp-brane'
  - Filling (p+1)-dim
  - Gauge theories appear on D-branes
  - D-branes wrapping over magnetized extra dims realize the previous situation.

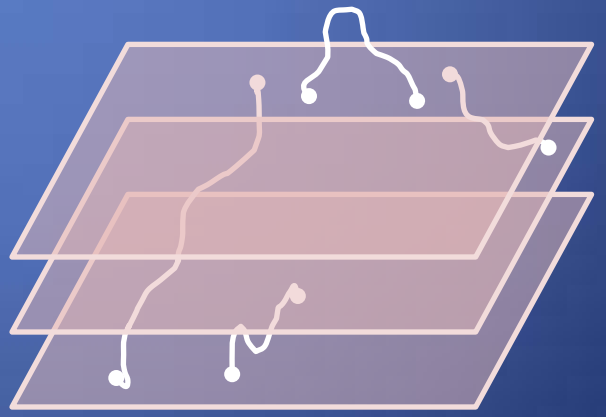
Magnetized branes





Magnetized branes



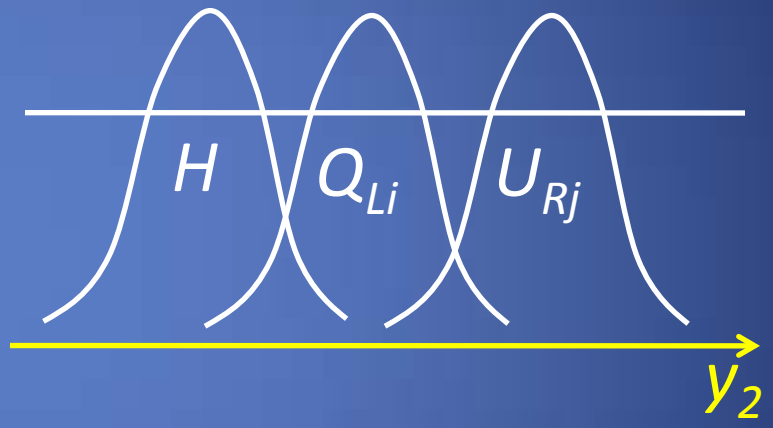


Dual description

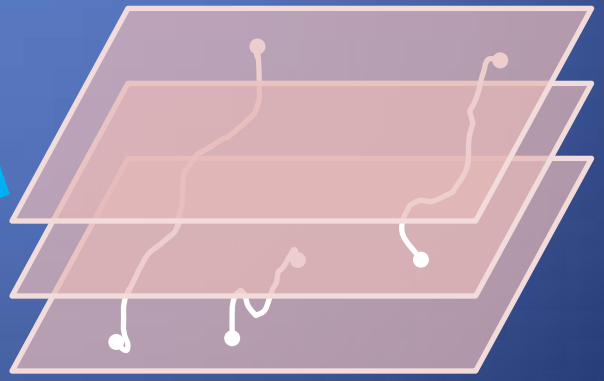
Localized modes come from strings localized at intersections

Intersecting branes

Magnetized branes



T-dual

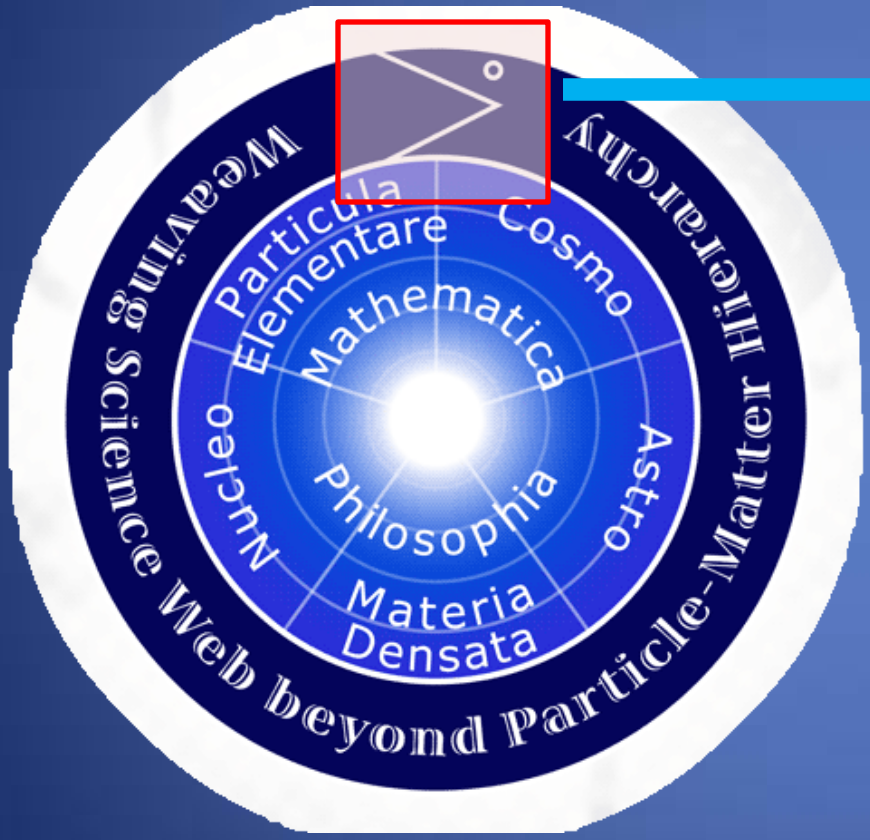


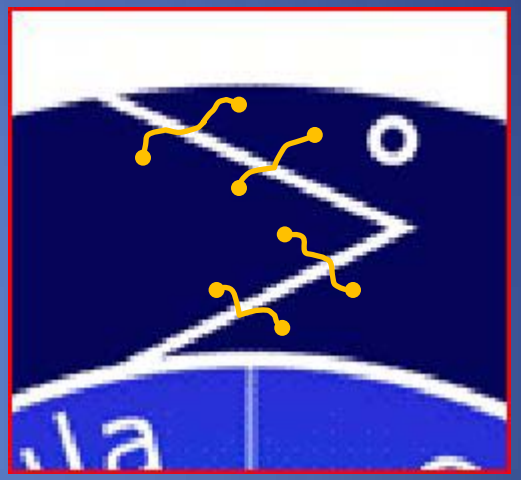
SUMMARY

Hierarchical elements of our world

- Not God but physics (dynamics) could yield the hierarchy of the elements.
- Hints were provided by a unification of elementary particles and gravity.
- Magnetized extra dims (orbifold)
  - Magnetic flux determines almost everything
    gauge syms, chirality, # of gens, hierarchies, …
  - Combination of flux & projection play a role
  - Low energy effective theory of superstrings

Science web





Realistic string models of elementary particles are desired

THANK YOU